# DOES JOINT MARKETING PROMOTIONS RESULT IN A PRISONERS DILEMMA? 

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#### Abstract

There is an existing phenomenon that firms tend to launch price promotions during the same period. Firms that use joint price promotions seem to be engaging in an irrational behavior, because consumers might misinterpret price variations as a reduction in quality. Secondly, a "prisoners' dilemma" outcome might occur as competitive members launch price promotions simultaneously. This study employs a Bertrand-Nash equilibrium model to demonstrate the risk reduction role of simultaneous price promotions, and shows how price discount campaigns result in a lower price competition among players.


Keywords: Collective Price Promotions, Bertrand Model, Advertising.

## INTRODUCTION

Collective price promotions, also known as cooperative price promotions, refers to multiple stores offer price discounts simultaneously. Although major department stores launch anniversary sales during the similar period, few literatures pay attention to these collective phenomena. Firms may offer price discounts to attract more customers in many circumstances, however, the literature has extensively documented the adverse effects of price promotions on long-term sales and brand equity (Rao \& Monroe, 1989; Dodds et al., 1991; Mayhew \& Winer, 1992; Erdem et al., 2008). Prior to the scanner data revolution in the 1980s, the ratio of advertising expenditure to price promotion in the U.S. was about 60:40. In modern time, many companies selling packaged consumer goods, price promotion accounts for about 70 percent of the marketing budget (Ailawadi et al., 2001). Price promotion is the largest single category in the marketing-budget mix of U.S. packaged goods companies (Silva-Risso et al., 1999). One explanation for these phenomena is that price promotions are an inevitable result of the prisoner's dilemma (Lal, 1990), where the gains from price discounts are significant if no one else makes a similar offer, and hence every manufacturer and retailer ends up offering similar deals. As products are more toward homogeneous, the firm does not participate the price promotions would lose market share. Relative game-theoretic analyses indicate that competitors can avoid this prisoner's dilemma by not promoting in the same period (Kinberg, 1974; Sobel, 1984; Lal, 1990), but this result cannot explain the fact that brands often offer price discounts simultaneously, for routine events such as anniversary promotions and trade shows. It seems odd that manufacturers and retailers keep rising the budgets on an activity that can jeopardize long-term brand equity. These practices are so commonly implemented that we must look again
at the question of whether joint price promotions are simply an outcome of the prisoner's dilemma. If collective price promotions are a result of fierce competition, the frequency and budget on them should decline in the presence of more concentrated industries.

This study uses a Bertrand model to explain the nature of collective price discounts in rational expectations equilibrium. When simultaneous price decline hurts both parties' profit, the coordinated activity creates a risk-reduction effect on the consumers' perception. Through joint price promotions, consumers realize that price adjustments are caused by regular events rather than quality downgrade. From companies' perspective, collective price promotion relives the negative effects from price promotions and enhances the positive side. Although our proposition that joint price promotion can increase collective profits may contradict traditional economic intuition, it nevertheless provides practical insights into these commonly used practices.

## RATIONAL EXPECTATIONS MODEL

The rational-expectations model was initially proposed by (Muth, 1961; Lucas, 1972) and being extended by (Grossman, 1976; Grossman \& Stiglitz; 1980). Hellwig (1980) argued that since noise comes from the supply side, the price cannot provide sufficient information, and so simply observing price cannot provide enough information to predict the asset's return. His model was solved by (Admati, 1985), in which price can serve as exogenous and endogenous variable at the same time (Easley \& O'Hara, 2004).

Assume that a duopoly market in which there are two stores, each firm carries one good; the average quality of firm $i$ 's good is $\bar{q}$, the total cost function of firm $i$ is $T C_{i}=c z_{i}$, the unit cost of production is $c>0$, the quantity of goods is $z$, and subscript $i$ signifies firm 1 or 2 . Let $\lambda \in(0,1)$ be the fraction of consumers whose expectations regarding quality are based on both a good's advertising and its price, and let $1-\lambda$ be the fraction of consumers who infer quality only by observing the good's price.

Advertising serves the role of communicating quality with a white noise as follows:

$$
\begin{equation*}
\mathbf{A}=\mathbf{Q}+\boldsymbol{\varepsilon} \tag{1}
\end{equation*}
$$

Where $\mathbf{A}=\left[\begin{array}{ll}a_{1} & a_{2}\end{array}\right]^{\prime}$, where $a_{i}$ is the advertising information on firm $i$ 's good received by consumers? The advertising information reflects the actual quality, $\mathbf{Q}=\left[\begin{array}{ll}q_{1} & q_{2}\end{array}\right]^{\prime}$, where $q_{i}$ denotes the actual quality of firm $i$. As noise gets smaller, advertising could transmit signal of quality precisely. Assume that $\mathbf{Q}$ follows a normal distribution, where mean and variance can be expressed as follows:

$$
\mathbf{Q} \sim N(\bar{Q}, \mathbf{V}), \quad \bar{Q}=\left[\begin{array}{l}
\bar{q} \\
\bar{q}
\end{array}\right], \quad \mathbf{V}=\left[\begin{array}{ll}
v_{11} & v_{12} \\
v_{12} & v_{22}
\end{array}\right]
$$

The noises contained in advertising information are denoted by $\boldsymbol{\varepsilon}=\left[\begin{array}{ll}\varepsilon_{1} & \varepsilon_{2}\end{array}\right]^{\prime}$, which follows a normal distribution:

$$
\varepsilon \sim \mathrm{N}(\mathbf{0}, \mathbf{S}), \quad \mathbf{S}=\left[\begin{array}{ll}
s_{11} & s_{12} \\
s_{12} & s_{22}
\end{array}\right]
$$

Advertising information with a low $s_{i i}$ is more precise in communicating quality, so $s_{i i}^{-1}$ can behave as advertising precision. In order to underscore the advertising effect, we assume that only the good of firm 1 is advertised, while the good of firm 2 is not, it is equivalent to assume

$$
\begin{equation*}
s_{22} \rightarrow \infty \tag{2}
\end{equation*}
$$

The consumers' utility function is $\mathbf{u}(\mathbf{Q})=-\exp \left(-\gamma^{-1} \mathbf{Q}\right)$; given that $\mathbf{Q}$ follows a normal distribution, the demand of consumers that assess quality only by observing the retail price is

$$
\begin{equation*}
\mathbf{Z}_{\mathbf{d}}(\mathbf{P})=\gamma \operatorname{Var}(\mathbf{Q} \mid \mathbf{P})^{-1}(E(\mathbf{Q} \mid \mathbf{P})-\mathbf{P}) \tag{3}
\end{equation*}
$$

And the demand of consumers that assess quality both by advertising and price is

$$
\begin{equation*}
\mathbf{Z}_{d}(\mathbf{A}, \mathbf{P})=\gamma \operatorname{Var}(\mathbf{Q} \mid \mathbf{A}, \mathbf{P})^{-1}(E(\mathbf{Q} \mid \mathbf{A}, \mathbf{P})-\mathbf{P}) \tag{4}
\end{equation*}
$$

Where $\mathbf{P}=\left[\begin{array}{ll}p_{1} & p_{2}\end{array}\right]^{\prime}$ denotes the retail price and $\gamma>0$ is the level of risk tolerance.
This study assumes that a change in supply takes place at the same time as price promotion in order to guarantee equilibrium. The supply quantity is denoted by $\mathbf{Z}_{\mathrm{s}}$, which follows a normal distribution:

$$
\mathbf{Z}_{s} \sim N(\overline{\mathbf{Z}}, \mathbf{U}), \quad \overline{\mathbf{Z}}=\left[\begin{array}{l}
\bar{z}_{1} \\
\bar{z}_{2}
\end{array}\right], \quad \mathbf{U}=\left[\begin{array}{ll}
u_{11} & u_{12} \\
u_{12} & u_{22}
\end{array}\right]
$$

The supply variances $u_{11}$ and $u_{22}$ represent the intensity of price promotions on advertised good and unadvertised goods, describing that the manufacturer holds stocks of goods and then release them while offering discounts. In addition, $u_{12}$ represents the covariance between price promotions of two firms, with a higher $u_{12}$ meaning that the two firms hold collective price promotions more frequently.

## MARKET EQUILIBRIUM

Assume $\mathbf{Z}_{\mathbf{d}}$ be the average demand quantity by taking average of informed and uninformed consumer demand:

$$
\begin{equation*}
\overline{\mathbf{Z}}_{\mathbf{d}}=\lambda \gamma \frac{[E(\mathbf{Q} \mid \mathbf{A}, \mathbf{P})-\mathbf{P}]}{\operatorname{Var}(\mathbf{Q} \mid \mathbf{A}, \mathbf{P})}+(1-\lambda) \gamma \frac{[E(\mathbf{Q} \mid \mathbf{P})-\mathbf{P}]}{\operatorname{Var}(\mathbf{Q} \mid \mathbf{P})} \tag{5}
\end{equation*}
$$

Let $\overline{\mathbf{Z}}_{\mathbf{d}}=\mathbf{Z}_{\mathrm{s}}$, we obtain an equilibrium price matrix of advertising and output:

$$
\begin{equation*}
P=B_{0}+B_{1} A-B_{2} Z_{s} \tag{6}
\end{equation*}
$$

Where

$$
\begin{gathered}
\mathbf{B}_{0}=\mathbf{W}_{0}\left[\gamma^{-1}\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1} \overline{\mathbf{Z}}+\mathbf{V}^{-1} \overline{\mathbf{Q}}\right], \\
\mathbf{B}_{1}=\mathbf{W}_{0}\left[\lambda \mathbf{S}^{-1}+\left(\mathbf{S}+\lambda^{-2} \gamma^{-2} \mathbf{S U S}\right)^{-1}\right], \\
\mathbf{B}_{2}=\gamma^{-1} \mathbf{W}_{0}\left[\mathbf{I}+\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1}\right], \\
\mathbf{W}_{\mathbf{0}}=\left[\lambda \mathbf{S}^{-1}+\mathbf{V}^{-1}+\left(\mathbf{S}+\lambda^{-2} \gamma^{-2} \mathbf{S} \mathbf{U S}\right)^{-1}\right]^{-1},
\end{gathered}
$$

Please see derivation of Equation (6) in (Admati, 1985). In Equations (3)-(4), the price serves as an exogeneous variable when consumers see it as an indication of quality. In Equations (5)-(6), price being solved becomes an endogenous variable. The property not only matches consumer behavior but also allows us to conduct a numerical analysis. As previous noted, in order to underscore the advertising effect, we assume that the good of firm 1 uses advertising, while the good of firm 2 does not. Take expectation of equation (6), we can obtain the prices that can be analyzed as a demand function (Ozdenoren \& Yuan, 2008).

Lemma 1: Suppose that the good carried by firm 1 is advertised, while the good carried by firm 2 is not. The demand functions $p_{i}^{e}\left(z_{1}, z_{2}\right), i=1,2$ for an advertised and an unadvertised good are:

$$
\begin{align*}
& p_{1}^{e}=\bar{q}-b_{11} z_{1}-b_{12} z_{2}  \tag{7}\\
& p_{2}^{e}=\bar{q}-b_{12} z_{1}-b_{22} z_{2} \tag{8}
\end{align*}
$$

Where

$$
\begin{gathered}
b_{11}=\frac{v_{11} \gamma}{\left(v_{11} v_{22}-v_{12}^{2}\right)}\left[\left(\frac{\lambda \gamma}{s_{11}}+\frac{u_{22} \gamma}{s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)}+\frac{v_{22} \gamma}{v_{11} v_{22}-v_{12}^{2}}\right)\left(\frac{v_{11} \gamma}{v_{11} v_{22}-v_{12}^{2}}\right)-\frac{v_{12}^{2} \gamma^{2}}{v_{11} v_{22}-v_{12}^{2}}\right]^{-1} \\
b_{12}=\frac{v_{12} \gamma}{\left(v_{11} v_{22}-v_{12}^{2}\right)}\left[\left(\frac{\lambda \gamma}{s_{11}}+\frac{u_{22} \gamma}{s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)}+\frac{v_{22} \gamma}{v_{11} v_{22}-v_{12}^{2}}\right)\left(\frac{v_{11} \gamma}{v_{11} v_{22}-v_{12}^{2}}\right)-\frac{v_{12}^{2} \gamma^{2}}{v_{11} v_{22}-v_{12}^{2}}\right]^{-1} \\
b_{22}=\left[\frac{v_{11} \gamma}{v_{11} v_{22}-v_{12}^{2}}-\frac{v_{12}^{2} \gamma^{2}}{v_{11} v_{22}-v_{12}^{2}}\left(\frac{\lambda \gamma}{s_{11}}+\frac{u_{22} \gamma}{s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)}+\frac{v_{22} \gamma}{v_{11} v_{22}-v_{12}^{2}}\right)^{-1}\right]^{-1} .
\end{gathered}
$$

Proof: The expected value of equation (6) is

$$
\begin{aligned}
& E(\mathbf{P})=E\left(\mathbf{B}_{\mathbf{0}}\right)+E\left(\mathbf{B}_{\mathbf{1}} \mathbf{A}\right)-E\left(\mathbf{B}_{\mathbf{2}} \mathbf{Z}_{\mathbf{s}}\right) \\
& =\mathbf{W}_{\mathbf{0}}\left[\gamma^{-1}\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1} \overline{\mathbf{Z}}+\mathbf{V}^{-1} \overline{\mathbf{Q}}\right]+\mathbf{W}_{\mathbf{0}}\left[\lambda \mathbf{S}^{-1}+\left(\mathbf{S}+\lambda^{-2} \gamma^{-2} \mathbf{S} \mathbf{U S}\right)^{-1}\right] \overline{\mathbf{Q}} \\
& +\mathbf{W}_{\mathbf{0}} \gamma^{-1}\left[\mathbf{I}+\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1}\right] \overline{\mathbf{Z}} \\
& =\mathbf{W}_{\mathbf{0}}\left[\mathbf{V}^{-1}+\lambda \mathbf{S}^{-1}+\left(\mathbf{S}+\lambda^{-2} \gamma^{-2} \mathbf{S U S}\right)^{-1}\right] \overline{\mathbf{Q}}+\mathbf{W}_{0} \gamma^{-1}\left[\mathbf{I}+\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1}-\left(\lambda+\lambda^{-1} \gamma^{-2} \mathbf{U S}\right)^{-1}\right] \\
& =\mathbf{W}_{\mathbf{0}} \mathbf{W}_{\mathbf{0}}^{-1} \overline{\mathbf{Q}}+\mathbf{W}_{\mathbf{0}} \boldsymbol{\gamma}^{-1} \overline{\mathbf{Z}}=\overline{\mathbf{Q}}+\mathbf{W}_{\mathbf{0}} \boldsymbol{\gamma}^{-1} \overline{\mathbf{Z}} .
\end{aligned}
$$

Then let $s_{2} \rightarrow \infty$ and expand $\mathbf{W}_{\mathbf{0}} \gamma^{-1}$, and we can solve $b_{11}, b_{12}$ and $b_{22}$.
Q.E.D.

## BERTRAND MODEL

Bertrand model serves an example of price war between two duopoly firms who offer price discounts collectively. First, we solve the demand functions in (7) and (8) to generate the following demand functions:

$$
\begin{align*}
& z_{1}=\frac{\bar{q}\left(b_{22}-b_{12}\right)}{b_{11} b_{22}-b_{12}^{2}}-\frac{b_{22}}{b_{11} b_{22}-b_{12}^{2}} p_{1}^{e}+\frac{b_{12}}{b_{11} b_{22}-b_{12}^{2}} p_{2}^{e}  \tag{9}\\
& z_{2}=\frac{\bar{q}\left(b_{11}-b_{12}\right)}{b_{11} b_{22}-b_{12}^{2}}+\frac{b_{12}}{b_{11} b_{22}-b_{12}^{2}} p_{1}^{e}-\frac{b_{11}}{b_{11} b_{22}-b_{12}^{2}} p_{2}^{e} \tag{10}
\end{align*}
$$

Where the expressions of $b_{11}, b_{12}$ and $b_{22}$ are given in Equation (7) and (8).The choice variable is price in the Bertrand model, the equilibrium solution turns out to be the same as the Nash equilibrium. We derive each firm's first-order condition with respect to its own price for a given rival price and solve the system of equations.

Lemma 2: Assume that two firms compete against another in a duopoly market. Firm 1 that holds the advertised good faces a demand function (9), Firm 2 that holds the unadvertised good faces a demand function (10), the total cost function of firm $i$ is $T C_{i}=c z_{i}, i=1,2$. The optimal prices of firm $i$ in Bertrand-Nash equilibrium are given as follows.

$$
\begin{align*}
& p_{1}^{*}=\frac{\bar{q}\left(2 b_{11} b_{22}-b_{11} b_{12}-b_{12}^{2}\right)+c\left(2 b_{11} b_{22}+b_{11} b_{12}\right)}{4 b_{11} b_{22}-b_{12}^{2}}  \tag{11}\\
& p_{2}^{*}=\frac{\bar{q}\left(2 b_{11} b_{22}-b_{22} b_{12}-b_{12}^{2}\right)+c\left(2 b_{11} b_{22}+b_{22} b_{12}\right)}{4 b_{11} b_{22}-b_{12}^{2}} \tag{12}
\end{align*}
$$

Proof: Firm $i$ 's profit can be written as:

$$
\begin{equation*}
\pi_{i}=\left(p_{i}^{e}-c\right) \cdot z_{i}=\left(p_{i}^{e}-c\right)\left[\frac{\bar{q}\left(b_{22}-b_{12}\right)}{b_{11} b_{22}-b_{12}^{2}}-\frac{b_{i j}}{b_{11} b_{22}-b_{12}^{2}} p_{i}^{e}+\frac{b_{i j}}{b_{11} b_{22}-b_{12}^{2}} p_{j}^{e}\right], i, j=1,2 \tag{7}
\end{equation*}
$$

Solving two functions $\partial \pi_{i} / \partial p_{i}^{e}=0$ simultaneously gives equations (9) and (10).
Q.E.D.

A way to increase the prices and profits in the Bertrand model is use price promotions in the same period. We show that an increase in covariance of price promotions ( $u_{12}$ ) can lead to an increase in both firms' profits and reduce price competition, the effect of intensity of price promotions to profits and price competition is similar.

Proposition 1: Given that covariance between two goods' quality is lower than variance of two goods' quality ( $v_{12}<v_{11}, v_{12}<v_{22}$ ) and the expected quality is larger than unit cost $(\bar{q}>c)$, the price competition between advertised good and unadvertised good in Bertrand-Nash equilibrium decreases with an increase in covariance of price promotions ( $u_{12}$ ), and the profits for both firm in Bertrand-Nash equilibrium increase with an increase in covariance of price promotions ( $u_{12}$ ).
Proof: The prices and output of advertised good and unadvertised good can be given in terms of $\alpha, \beta$ and $\theta$ :

$$
\begin{align*}
& p_{1}^{*}=\frac{\bar{q}\left(2 \alpha \beta-\beta \theta-\theta^{2}\right)+c(2 \alpha \beta+\beta \theta)}{4 \alpha \beta-\theta^{2}}  \tag{13}\\
& p_{2}^{*}=\frac{\bar{q}\left(2 \alpha \beta-\alpha \theta-\theta^{2}\right)+c(2 \alpha \beta+\alpha \theta)}{4 \alpha \beta-\theta^{2}}  \tag{14}\\
& z_{1}^{*}=\frac{(\bar{q}-c)\left(2 \alpha^{2} \beta-\alpha \beta \theta-\alpha \theta^{2}\right)}{4 \alpha \beta-\theta^{2}}  \tag{15}\\
& z_{2}^{*}=\frac{(\bar{q}-c)\left(2 \alpha \beta^{2}-\alpha \beta \theta-\beta \theta^{2}\right)}{4 \alpha \beta-\theta^{2}} \tag{16}
\end{align*}
$$

Where

$$
\begin{gathered}
\alpha=\frac{\lambda \gamma}{s_{11}}+\frac{u_{22} \gamma}{s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)}+\frac{v_{22} \gamma}{v_{11} v_{22}-v_{12}^{2}}, \\
\beta=\frac{v_{11} \gamma}{\left(v_{11} v_{22}-v_{12}^{2}\right)}, \\
\theta=\frac{v_{12} \gamma}{\left(v_{11} v_{22}-v_{12}^{2}\right)},
\end{gathered}
$$

Then we differentiate $\alpha$ with respect to $u_{11}$, to get

$$
\begin{aligned}
& \frac{\partial \alpha}{\partial u_{11}}= \frac{-\lambda^{-2} \gamma^{-1} s_{11}^{2} u_{22}^{2}}{\left(s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)\right)^{2}}<0 \\
& \frac{\partial \alpha}{\partial u_{22}}=\frac{-\lambda^{-2} \gamma^{-1} s_{11}^{2} u_{12}^{2}}{\left(s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)\right)^{2}}<0 \\
& \frac{\partial \alpha}{\partial u_{12}}=\frac{2 \lambda^{-2} \gamma^{-1} s_{11}^{2} u_{22} u_{12}}{\left(s_{11} u_{22}+(\lambda \gamma)^{-2} s_{11}^{2}\left(u_{11} u_{22}-u_{12}^{2}\right)\right)^{2}}>0
\end{aligned}
$$

Since $\partial \alpha / \partial u_{12}>0$, the assumptions $\bar{q}-c>0$ and $v_{12}<v_{11}, v_{12}<v_{22}($ so $\theta<\alpha, \theta<\beta)$ ensure that the following differentiations are all positive:

$$
\begin{align*}
& \frac{\partial\left(p_{1}^{*}-c\right)}{\partial u_{12}}=\frac{(\bar{q}-c)\left(4 \beta^{2} \theta+2 \beta \theta^{2}\right)}{\left(4 \alpha \beta-\theta^{2}\right)^{2}} \frac{\partial \alpha}{\partial u_{12}}>0  \tag{17}\\
& \frac{\partial z_{1}^{*}}{\partial u_{12}}=\frac{(\bar{q}-c)\left(8 \alpha^{2} \beta^{2}-4 \alpha \beta \theta^{2}+\beta \theta^{3}+\theta^{4}\right)}{\left(4 \alpha \beta-\theta^{2}\right)^{2}} \frac{\partial \alpha}{\partial u_{12}}>0  \tag{18}\\
& \frac{\partial\left(p_{2}^{*}-c\right)}{\partial u_{11}}=\frac{(\bar{q}-c)\left(2 \beta \theta^{2}+\theta^{3}\right)}{\left(4 \alpha \beta-\theta^{2}\right)^{2}} \frac{\partial \alpha}{\partial u_{11}}>0  \tag{19}\\
& \frac{\partial z_{2}^{*}}{\partial u_{12}}=\frac{(\bar{q}-c)\left(2 \beta^{2} \theta^{2}+\beta \theta^{3}\right)}{\left(4 \alpha \beta-\theta^{2}\right)^{2}} \frac{\partial \alpha}{\partial u_{12}}>0 \tag{20}
\end{align*}
$$

The difference of $p_{1}^{*}$ and $p_{2}^{*}$ obtained from (13) and (14) increases with $u_{12}$ :

$$
\begin{equation*}
\frac{\partial\left(p_{1}^{*}-p_{2}^{*}\right)}{\partial u_{12}}=\frac{\partial\left[\frac{\theta(\bar{q}-c)(\alpha-\beta)}{\left(4 \alpha \beta-\theta^{2}\right)}\right]}{\partial u_{12}}=\frac{\theta(q-c)\left(4 \beta^{2}-\theta^{2}\right)}{\left(4 \alpha \beta-\theta^{2}\right)^{2}} \frac{\partial \alpha}{\partial u_{12}}>0 \tag{21}
\end{equation*}
$$

Q.E.D.

Proposition 1 show that the profits of an advertised and an unadvertised good are both lifted with an increase in covariance of price movements. The price competition is lowered in the presence of these concerted actions; the relief in risk perception contradicts the traditional wisdom, which indicates that joint price promotion will result in a fiercer price competition. Although price promotions lower consumers' quality perception, joint price promotions relieve concern over quality because consumers would realize the price discounts are not results of inferior quality but of a regular event.

## CONCLUSION

Traditional theory indicates that price promotions cause adverse effects on product image, so that firms should avoid engaging into price wars by offering price promotions alternatively. This study proposes a Bertrand model in which advertising and covariance of promotions cause a positive effect to the degree of product differentiation. While price promotion increases price competition and reduces individual profits, joint price promotions lower price competition and increase collective profits due to the role of covariance. The economic insight is that consumers will not misinterpret price promotions as an indicator of inferior quality because they are offered as a regular event participated by differentiated firms.

The contribution of this study explains why joint price promotions such as anniversary sales are commonly used. Under the traditional wisdom, joint price promotions can be seen as a form of fierce price competition caused by rival retaliation. A rational expectations model shows that joint price promotions can serve a risk reduction role; because consumers realize that price promotions are not necessarily a result of low quality. In addition, this study identifies the importance of mass retailers, which can coordinate joint price promotions for the brands they carry. Joint price promotions arranged by a mass retailer can increase the collective profits of manufacturers and in turn, the mass retailer's own benefit. Therefore, a mass retailer can serve the common interests of members under its umbrella by reducing consumers' perceived risk. This study also indicates the importance of advertising because the risk reduction role of joint promotions cannot exist without an advertised good. The combination of advertising spillover effect and joint price promotions, which can be coordinated by a mass retailer, can benefit unadvertised goods as well. Unadvertised, nondescript goods presented in joint price promotions benefit from the risk-reduction effect of simultaneously activities that is enhanced by advertised goods. The advertised goods also benefit from the facts they are integral and essential parts of retailers' holding portfolio that makes joint price promotions possible.

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