AN APPLICATION OF LINEAR PROGRAMMING IN PERFORMANCE EVALUATION

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ABSTRACT

Assessing performance of employees is an important process for achieving excellence. In many, evaluation processes, an employee's annual composite score is a weighted average of performance scores on several roles and weights chosen by the employee within bounds stipulated by the administration for each role. In this process, the composite score depends not on merit alone, but also on employee's choice of weights. In this paper, we propose a modified process-based on Linear Programming (LP) - that assigns to employees the optimal weights that are compatible with their supervisor-assigned scores in each role. The LP model is designed to assign role weights, within predefined ranges, such that the composite score is maximized. The overall score depends on the supervisor's assessment of performance alone, eliminating the need for the employee to "correctly" choose weights. The modified approach should lead to more valid evaluations of performance and improved employee satisfaction with the annual performance review process.

Since the associated LP problem has relatively few constraints, we solved it explicitly and implemented the solution in a smart pdf form for use by supervisors. LP codes are usually implemented in expensive commercial software. We believe that this is the first time that such a code has been successfully embedded in a smart pdf form.

Keywords: Linear Programming, Evaluation Processes, Smart pdf Form

INTRODUCTION

Assessing performance of employees is an important process for achieving excellence. Performance evaluations serve as a basis for many key decisions such as compensation, promotion, and employee development. Organizations that lack effective evaluation systems may experience higher rates of employee dissatisfaction, attrition, and lowered productivity. Universities and colleges, like other organizations, are expected to develop effective systems for evaluating faculty performance. There are some unique challenges to evaluating faculty performance because of the wide-ranging activities in which faculty regularly engage. Never the less, a consensus has emerged on the need to establish valid and reliable systems of assessing faculty performance (Wolfer & Johnson, 2003).

The distribution of responsibilities and allocation of effort to various components of responsibility such as teaching, research, and service affects the ability to perform in each of these components (Ridley & Collins, 2015). It is therefore necessary to present a complete measure of performance while recognizing the relative importance of these various components. Systems of faculty evaluation and reward should recognize differing patterns of productivity in faculty as well as the mission of the institution (Boyer, 1990).

Elmore (2008) suggests that assessment of faculty work should be based on the most empirical and objective means possible. Miller & Seldin (2014) show that academic administrators are under growing pressure to assess faculty performance through formalized, systematic methods and that deans weigh a wide range of factors in the evaluation process. Practices of assigning difference weights to different roles vary widely among institutions and are influenced by numerous institutional characteristics (Centra, 1977).

In many performance evaluation processes, a supervisor assigns scores, subject to a predefined scale, for specified areas of responsibility. An overall score is determined by weighting the scores from the individual areas such that the weights account for 100% of the total effort. For example, a Department Chair assigns scores on a 0.0-4.0 scale to assess a faculty member's teaching effectiveness, student engagement, scholarship, and service. These scores are then weighted, based on the level of importance attached to each area, to obtain an overall evaluation score. In many cases, the faculty has some degree of flexibility in choosing the weights assigned to their areas of responsibility.

Caldwell Jr. & Schulte (2002) describe results of their survey showing various indicators of faculty dissatisfaction with the evaluation process. Indicators of dissatisfaction included uncertainty of specific responsibilities, difficulty in preparing for promotion and tenure because of a lack of consistency in performance evaluations, and a lack of standards for various aspects of their responsibilities. Allowing faculty to choose their own weights, within predefined ranges, gives them more freedom and control and should lead to greater satisfaction with the evaluation process. Arreola (2006) recommends the use of such a dynamic faculty role model rather than a static faculty role model.

Linear Programming (also called Linear Optimization or LP) is the study of methods to achieve an optimal outcome in a linear mathematical model. It uses mathematical techniques to find an optimal value for a linear objective function, subject to linear equality and/or inequality constraints. It was originally developed during the Second World War, mainly by George Dantzig, to optimize the use of limited (i.e. constrained) military resources. A historical treatment of the subject can be found in Lenstra, Rinnooy Kan & Schrijver (1991). However, it has since been extended to a wide variety of business, engineering and scientific applications (Charnes & Cooper, 1961; Gärtner & Matoušek, 2006; Dantzig &Thapa,1997). In this paper, we propose an application of the technique to the faculty performance evaluation process.

THE SCENARIO

At Georgia Gwinnett College, the faculty has some degree of flexibility in choosing the area evaluation weights, within predetermined intervals, at the time they submit their end-of-year evaluation portfolios. Some schools within the college have experimented with situations in which faculty members assign their area evaluation weights at the beginning of the academic year. However, the rigidity of this process tended to create problems for the faculty. For instance, a faculty member might assign higher weights to the areas of scholarship and student engagement, and lower weights to teaching and service at the beginning of the year, in anticipation of the achievement of higher outcomes in the first two areas and lower outcomes in the second two during the year. However, as is often the case, the faculty member may obtain better outcomes in the lesser-weighted areas and then end up with a low overall evaluation score, caused by the rigid pre-assigned unfavorable weights. This led to dissatisfaction and contributed to the abandonment of the practice.

Since faculty desire high evaluation scores, they tend to select their weights, while preparing their end-of-year evaluation portfolios, based on their perceptions of how well they performed in each evaluated area. For instance, a faculty member who feels that he or she performed well in the areas of scholarship and student engagement, but not so well in teaching and in service, would typically assign higher weights for the first two roles and lower weights to the second two. However, if the supervisor's review of the faculty member's areas of strength and weakness differ from the faculty member's perception - as is often the case-the weights selected by the faculty member end up yielding an unfavorable overall evaluation score. Even in the case that the faculty member's self-assessment of areas of strength and weakness coincides with the assessment arrived at by the supervisor, it is virtually impossible for a faculty member to choose the right mix of weights that will lead to an optimal evaluation score.

This often leads to situations where faculty members play a numbers game, which creates doubts about the validity of evaluation process. Suppose, for instance, that a college assigns the following intervals, as shown in Table 1, for the weights of the four evaluated roles that we have been considering.

Table 1
WEIGHTS OF THE FOUR EVALUATED ROLES

Role	Minimum Weight	Maximum Weight
Teaching	45%	60%
Student Engagement	15%	30%
Scholarship	10%	30%
Service	10%	30%

Suppose that two fictitious faculty members, professors A and B selected their role weights within these intervals shown in Table 2 below.

Table 2
WEIGHT INTERVALS OF PROFESSOR A AND B

Role	Professor A Role Weights	Professor B Role Weights
Teaching	60%	45%
Student Engagement	15%	15%
Scholarship	12%	10%
Service	13%	30%

Additionally, suppose that the supervisor assigns both professors the following identical performance scores, on a scale of 0.0–4.0, shown in Table 3.

Table 3
IDENTICAL PERFORMANCE SCORES OF A AND B

Role	Score
Teaching	**4.0**
Student	**3.6**
Scholarship	**3.0**
Service	**3.0**

Professor A's overall evaluation score is (4.0 x 0.60)+(3.6 x 0.15)+(3.0 x 0.12)+(3.0 x 0.13)=3.69. However, Professor B's overall score is (4.0x0.45)+(3.6x0.15)+(3.0x0.10)+(3.0x0.30)=3.54, which is considerably different from Professor A's score. Several questions arise immediately. How fair is this process? Is this process really rewarding performance-which appears to have been identical for the two fictitious employees or is it rewarding their ability to choose favorable weights?

To resolve these problems, we have proposed, in our College, a modified evaluation process in which faculty do not choose their own weights, but instead, have optimal weights assigned automatically via Linear Programming in such a way that these weights fit within the assigned limits and maximize the overall performance score corresponding to their supervisor-assigned scores in each of the evaluated roles. The process identifies automatically the areas in which faculty members were most productive and rewards them in the most efficient possible way in those areas. It levels the playing field, by always assigning identical evaluation scores to any two faculty members that get identical performance scores, and by assigning an evaluation score that does not depend on a match between the faculty member's prior self-assessment and the supervisor's assessment.

We implemented our idea by designing a fillable pdf file for supervisors at Georgia Gwinnett College in which the scripts for the calculation of optimal weights are incorporated automatically using a relatively small number of lines of code, based on an explicit LP solution formula that we derived, as described in the appendix. LP codes are usually quite long, and are typically implemented in expensive commercial software. We believe that our smart form is the first instance in which an LP code has been successfully embedded in a smart fillable pdf form. Figure 1 shows the annotated evaluation summary illustrating the process flow of the smart form with fictitious data.

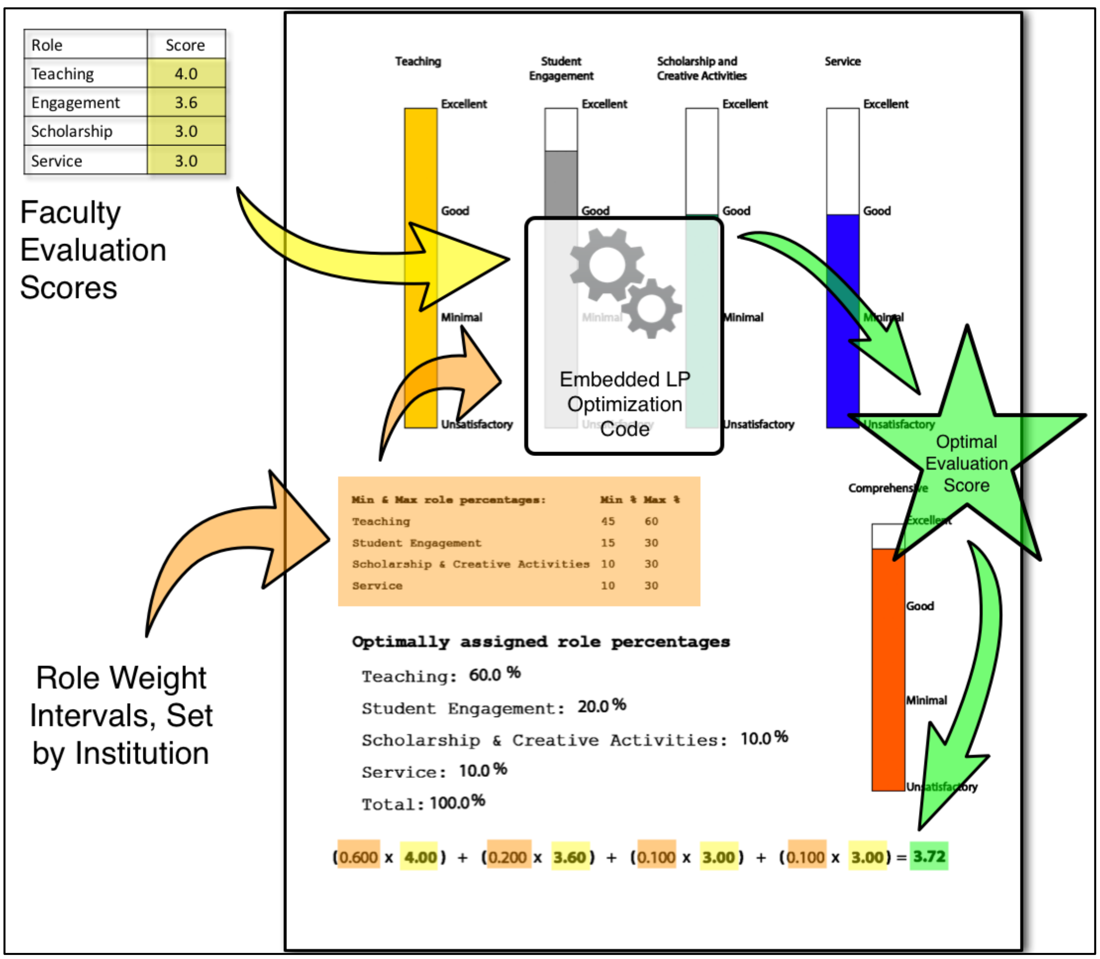

FIGURE 1
ANNOTATED EVALUATION SUMMARY ILLUSTRATING PROCESS FLOW

After reading the relevant portfolio and/or having a face-to-face meeting, a supervisor enters role scores for a faculty member into the smart form. The form then generates optimal weights for the evaluated roles, within the pre-assigned limits, and uses them to calculate composite overall scores and display bar charts of employees' role scores and overall composite scores.

Our form was considered by the Georgia Gwinnett College Faculty Senate in the spring of 2017 and given an overwhelming endorsement. This gives credence to the supposition that the adoption of the method may lead to improved faculty satisfaction with the evaluation process.

APPLICATION OF THE LP MODEL

The algorithm (see the Appendix) finds the explicit solution of the following LP problem:

$$\left.\begin{array}{c}\text{Maximize } c_1x_1+\cdots+c_nx_n \text{ subject to the constraints:}\\ a_i \leq x_i \leq b_i, i=1,\ldots,n, x_1+\cdots+x_n=1.\end{array}\right\}$$

It finds the optimal weights $x_1,\ldots,x_n$ satisfying the constraints, and the optimal objective function score $c=\sum_{i=1}^{n} c_i x_i$ corresponding to the pre-assigned 'scores' $c_1,\ldots,c_n$. Although we consider four roles for purposes of this paper, the model can be used to design a smart form for the evaluation of employees in n evaluated areas $R_1,\ldots,R_n$.

CONCLUSION

At our institution, faculty members are evaluated on four roles annually, namely Teaching, Student Engagement, Scholarship and Creative Activities, and Service. The performance evaluation process that we proposed in this paper assigns to each employee the optimal weights that are compatible with their supervisor-assigned scores in each role. We implemented our idea by designing a fillable pdf file for supervisors in which scripts for the calculation of optimal weights are incorporated automatically using a relatively small number of lines of code, based on the LP explicit solution formula derived previously. After reading employees portfolios and/or having face-to-face meetings, supervisors enter role scores for employees into the smart form. The form then generates optimal weights and uses these to calculate composite overall scores and display bar charts of employees' role scores and overall composite scores.

Because of this, overall evaluation scores depend on employees' performance alone rather than their ability or luck in choosing ideal role weights. We believe that this modified process levels the playing field and will lead to improved satisfaction with the annual performance review process.

It is also quite easy to implement the proposed evaluation process in a smart EXCEL® performance evaluation form or simply within an executable form on a web page. However, we preferred a portable document format (pdf) implementation because it is easier to use a pdf form in a standalone manner. It is very easy to extract data from and provide automation within a pdf form, and the resulting weights and scores are automatically entered into the employee's record.

Modified versions of our smart form can be used to provide cost-effective autonomous solutions in a wide variety of modeling situations. Suppose for instance that a company wishes to assign different fractional totals $x_1,\ldots,x_n$ of a product to n retail outlets $R_1,\ldots,R_n$, respectively, in such a way that $x_1+\cdots+x_n=1$. If the minimum and maximum capacity of the retail outlet R_i for this product is specified by an inequality of the form $a_i \leq x_i \leq b_i$, and c_i is the profit earned by selling one unit of the product at the outlet R_i, then our algorithm provides optimal values for the allocation of the quantities x_1,..., x_n. More elaborate examples of resource allocation problems can be found, for example, in King (1989).

APPENDIX

FORMULATION, SOLUTION, AND IMPLEMENTATION OF THE LINEAR PROGRAMMING MODEL

We suppose that each employee is evaluated on the roles $R_1,\ldots,R_n$, and that $x_1,\ldots, x_n$, respectively, are the weights assigned to them. Suppose that company policy stipulates that the weights must satisfy the constraints

$$a_i \leq x_i \leq b_i, \text{ i=1, . . .,n.}$$

and the equation

$$x_1 + \cdots + x_n = 1.$$

If are $c_1, \ldots, c_n$ the respective scores that the supervisor assigns to the employee on these roles., then the Linear Programming (LP) problem corresponding to an optimized weight assignment process is as follows:

$$\left.\begin{array}{c}\text{Maximize } c_1x_1 + \cdots + c_nx_n \\ \text{Subject to the Constraints:} \\ a_i \leq x_i \leq b_i, i = 1, \ldots, n \\ x_1 + \cdots + x_n = 1.\end{array}\right\} \quad (1)$$

Given concrete values of the input variables n, $a_1, \ldots, a_n, b_1, \ldots, b_n$ and $c_1, \ldots, c_n$, this LP problem with can be solved with EXCEL or any of the other large number of available LP software. However, since it has relatively few constraints, we will be able to solve it explicitly. Since $x_n = 1 - \sum_{i=1}^{n-1} x_i$, the LP problem can be written in the equivalent reduced LP form:

$$\left.\begin{array}{c}\text{Maximize } (c_1 - c_n)x_1 + \cdots + (c_{n-1} - c_n)x_{n-1} \\ \text{Subject to the Constraints:} \\ a_i \leq x_i \leq b_i, i = 1, \ldots, n \\ 1 - b_n \leq x_1 + \cdots + x_{n-1} \leq 1 - a_n.\end{array}\right\} \quad (2)$$

We first observe that if $\sum_{i=1}^{n} a_i > 1$ or $\sum_{i=1}^{n} b_i < 1$, then the LP problem (1) and the reduced LP problem (2) have no feasible solutions, and it is impossible for any employee to select weights as required by the administration. Therefore, we assume, in all that follows, that the following feasibility condition holds:

$$\sum_{i=1}^{n} a_i \leq 1 \leq \sum_{i=1}^{n} b_i. \quad (3)$$

For simplicity, we will also assume that

$$c_1 \geq c_2 \geq \cdots \geq c_n. \quad (4)$$

This can be enforced, in practice, by sorting the sequence $c_1, \ldots, c_n$ as the first step in the solution implementation algorithm. Since the coefficients of the linear objective function (2) in the reduced LP problem are all positive and non-increasing, we can obtain an optimal solution by simply choosing the maximum possible x_i values within the feasible region

$R = \{(x_1, \ldots, x_{n-1}): 1 - b_n \leq x_1 + \cdots + x_{n-1} \leq 1 - a_n \text{ } and \text{ } a_i \leq x_i \leq b_i \text{ } for \text{ } i = 1, \ldots, n-1\}$ starting at $x_{n-1}$and working our way down to x_1.

Let $X(n) = (b_1, \ldots, b_{n-1})$ and $s_{n-1} = \sum_{j=1}^{n-1} X_j(n) = \sum_{j=1}^{n-1} b_j$. Then, it follows from the feasibility condition (1) that $s_{n-1} \geq 1 - b_n$. If the extra condition $s_{n-1} \leq 1 - a_n$ also holds, then $X(n)$ is an optimal solution to the reduced LP problem.

Suppose that $X(m+1)$ and s_m have been defined, and that $s_m > 1 - a_n$. Then we define the subsequent lower terms $X(m) = (X_1(m), \ldots, X_{n-1}(m))$ and s_{m-1} by induction in the form

$$X_i(m) = \begin{cases} b_i \text{ if } i < m, \\ \max\{a_m, 1 - a_n + b_m - s_m\} \text{ if } i = m, \\ X_i(m+1) \text{ if } i > m, \end{cases}$$

and $s_{m-1} = \sum_{j=1}^{n-1} X_j(m)$.

It follows from these definitions that the condition $1 - a_n + b_m - s_m \geq a_m$ holds if and only if $X_m(m) = 1 - a_n + b_m - s_m$ and $s_{m-1} = s_m + X_m(m) - b_m = 1 - a_m$. The condition $1 - a_n + b_m - s_m < a_m$ holds if and only if $X_m(m) = a_m$ and $s_{m-1} = s_m + a_m - b_m > 1 - a_n$. We observe that $s_m \geq 1 - a_m$ in either case, and that $s_{m-1} \leq s_m \leq s_{n-1}$.

The conditions $s_{m-1} > 1 - a_n$ hold for all $1 \leq m \leq n$ if and only if $X_m(m) = a_m$ for all $1 \leq m \leq n$. This is the case if and only if $s_{n-1} = b_1 + \cdots + b_{n-2} + b_{n-1} \geq s_{n-2} = b_1 + \cdots + b_{n-2} + a_{n-1} \geq \cdots \geq s_0 = a_1 + \cdots + a_{n-2} + a_{n-1} > 1 - a_n$, in contradiction to the feasibility condition (1). It follows that there must exist a largest index value p such that $1 \leq p \leq n$ and $s_{p-1} \leq 1 - a_n$. Since the conditions $s_{p-1} \geq 1 - a_n$ always holds, we see that $1 - b_n \leq s_{p-1} = 1 - a_n$. This implies that $X(p)$ is an optimal solution to the reduced LP problem. On setting $X_n(p) = 1 - \sum_{j=1}^{n-1} X_j(p)$, we obtain the explicit solution

$(x_1, \ldots, x_{n-1}, x_n) = (X_1(p), \ldots, X_{n-1}(p), X_n(p))$ to the initial LP problem.

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