AN APPLICATION OF TAGUCHI L9 METHOD IN BLACK SCHOLES MODEL FOR EUROPEAN CALL OPTION

Amir Ahmad Dar, B S Abdur Rahman Crescent Institute of Science and Technology
Anuradha N, B S Abdur Rahman Crescent Institute of Science and Technology

ABSTRACT

The Black Scholes Model (BSM) is an important tool in financial economics in order to measures the option price at some future date at some defined price. This study is based on the design of experiment (DOE) on option pricing model at one period. DOE is a set of relations between the inputs and outputs variables. The Taguchi's orthogonal array design is based on a mathematical model of factorial designs developed by R.C. Bose. This study is based on an application of Taguchi orthogonal array L9, in which the four parameters of BSM for European call option, is varied at three different levels. The aim of the experiment is to get the more and maximum realistic information regarding the input and output variables. The four parameters of BSM at one period are; the underlying asset price S_0*, the strike price* K*, the interest rate* r *and the volatility* σ*. The main aims of this study are:* ***a)*** *to determine which parameter impacts more or less on European call option at one period,* ***b)*** *the percentage contribution of each parameter and* ***c)*** *it discusses the section of the parameters for obtaining the best combination* ***d)*** *and whether the value of a call option follows a certain distribution. The different statistical tools being used in this paper are ANOVA, ANOM, Signal-to-Noise (S/N) ratio and Tukey Method.*

Keywords: BSM, Taguchi method, ANOM, S/N ratio, ANOVA, Tukey Method

INTRODUCTION

European call option is a derivative that gives the keeper the rights but not obligation to buy the underlying asset at the defined price at expiry date. In order to estimate the value of call option at time t=0 the Black Scholes formula is used. Black and Scholes developed a formula for European option without dividend paying in 1973. The parameters that are necessary to estimate the value of European call option using Black Scholes formula at one period are: the underlying asset price S_0, the strike price K, the interest rate r and the volatility σ. Trials are used in firms to investigate the factors that have the highest/lowest impact on response variable. For an owner of a European call option, it is better to know which input parameters effects more/less on call option. In order to know about the input parameters effects on response variable, the experimental design is carried out using orthogonal array.

The DOE using the Taguchi orthogonal array approach can economically satisfy the needs of solving problem and products design optimization projects. By applying the Taguchi's method researchers, engineers and scientists can reduce the time, resources and money required for little experimental investigation. The DOE gives the relation between the input and output variables. The DOE using the Taguchi Orthogonal array approach requires a good planning, wise arrangement of the experiments, and analysis expert of results. The

Taguchi's method became the standard method for DOE application. It was developed by Dr Genechi Taguchi (a Japanese engineer, 1940). It became one of the best DOE for scientists and engineers. Its main goal is to study the whole process parameters with only minimum balanced trials, called orthogonal array. The main aim of DOE (Taguchi's method) is to look which parameter (input) effects more on the response variable. It is a well-known, powerful and unique approach to product quality improvement.

There are two main approaches to DOE, Full Factorial design (FFD), and the Taguchi's method. FFD is a set of an experiment whose design consists of more than one factors each with discrete possible level and whose experiment units takes all possible combinations of all those levels across all such factors. For example, if there are K factors each at 3 levels, FFD has 4^K runs. This for 4 factors at 3 levels it would take 81 trials runs. The Taguchi method is a statistical tool developed by Genier Taguchi (1940's) a Japanese engineer, proposed a model for experiment design. The Taguchi experiment array design is using to arrange the parameters affecting the process and the levels of which they should be varied. Instead of having all possible experiment like FFD, Taguchi model provides a minimum number of experiments. In case of 4 factors and 3 levels, it would take 9 trials runs. The experiments are not randomly generated but they are based on judgmental sampling. It reduces time, resources and cost.

In this paper, the Taguchi method is using in order to do an experiment on European call option at one period. The paper looks at the effects of four parameters in the BSM-European call option at one period, the underlying asset price S_0, the strike price K, the interest rate r and the volatility σ. The Taguchi's orthogonal array L9(3^4) is used in order to estimate the factors that influence the performance criteria and also which factors are more important than others. The Analysis of Mean (ANOM), S/N ratio, Tukey Method and Analysis of variance (ANOVA) is used in order to get the objectives of this paper. In order to develop a DOE by using Taguchi method. The following points are necessary:

1. Define the response variable, in this study the response variable is the value of the European call option.
2. Select the input variables, there are four input variables that are: underlying asset, strike price, interest rate, and volatility in order to estimate the value of call option.
3. Select the number of levels, in this study we choose three levels as shown in table 1.
4. Select the orthogonal array, it is based on the 2^{nd} and 3^{rd} point. In this study we want to conduct an experiment in order to understand the influence of four independent with each having three set values on a call option, then L9 orthogonal array might be the right choice. It allows us to consider a selected subset of combinations of multiple factors at multiple levels.
5. Assigning the four independent variables to each column
6. Conduct the experiment
7. Analysis the data (the call option)

Therefore, it is necessary for an option owner to know which parameter effects more and how much it effects. In order to know about it, the ANOM describes the best combinations of the parameters where the value of European call option gets maximum and also it identify which parameter effects more on the call option. The ANOVA used in order to measure the percentage contribution of each independent on the call option.

Literature review

Now a day's Taguchi's method is using in every field such as producing planning, education, service system, Software testing (Lazic, 2013), Environmental Engineering, Biotechnology etc.

The Taguchi's statistical method is a methodical method for expansion of various factors with considering to production, cost, and quality. It offers the standard quality of a product is estimated by standard characterizes such as; larger is better, nominal is the best, smaller is better (Phadke, 1989; Roy, 2001; Phadke, 1998).

Park et al., (2017) explored the effects of material and processing conditions; the parameters are injection temperature, powder size, shear rate, and initial powder volume fraction (IPVF) on powder binder separation (PBS) in powder injection molding. It was found that IPVF is the most significant factor for the PBS by using Taguchi L9 method.

Celik et al., (2018) used the Taguchi approach in order to investigate the effects of different parameters on pressure loss and heat transfer by using the Taguchi approach and ANOVA. The different parameters are width, pitch, thickness and Reynolds numbers. It was found that the effects of width, pitch, thickness and Reynolds numbers are 29.96%, 4.7%, 1.73% and 60.61% respectively. It is clearly shown that the Reynolds numbers effect more on pressure loss and heat transfer.

Madhavi et al., (2017) investigated the effects of four parameters (speed, feed, depth of cut and material) on hardness and surface roughness. The Taguchi statistical L9 approach and ANOVA were used in order verify which parameters effects more on hardness and surface roughness. It was investigated that the speed affects less and material affects more on hardness and surface roughness.

Babu et al., (2017) investigated the effects of parameters such as cutting speed (CS), depth of cut (DC), and feed rate (FR) on tool life (TL). The Taguchi statistical approach, ANOVA and Signal-to-noise (S/N) ratio were used in order to identify which factor effects more on TL. It was investigated that the FR affects more and DC affects less on the TL.

Katata-Seru et al., (2017) used the Taguchi statistical approach and emulsification method in order to prepare Garlic essential oil nanoemulsions (GEON). Also investigated which factor effects more in GEON polydispersity index (PDI) and GEON droplets size. The factors are-oil-surfactant mixing ratio (OSMR), type of surfactants (TOS), homogenizer and surfactant concentrations (SC). It was found that TOS has a significant effect on the GEON PDI and GEON droplets size during the GEON preparation by using the Taguchi L9 approach.

Rao and Padmanabhan (2012) used the Taguchi's L9 orthogonal array design that how parameters (voltage, electrolyte and feed rate) concentration on MRR (metal removal rate). The ANOVA, S/N ratio, and Regression analysis have been used in order to analyze the effects of these parameters. The feed rate affects more on MMR.

Dar and Anuradha (2018) and Dar et al., (2018) used Taguchi orthogonal L9 and L27 array, ANOM and ANOVA in order to identify which parameter affects more on the probability of default. It was identified that the volatility affects more on the probability of default. See more References: (Kacker et al. 1991, Lee et al. 2003, Lee and Shin et al. 2003, Yang and Tarng 1998, Shangi 2002, Wang and Feng 2002, Thomas 2008, Silva et al. 2014, Shravani et al. 2011, Chan et al. 2012, Athereya and Venkatesh 2012).

The aim of the DOE is to get the more and maximum realistic information. A large number of variables demand a large number of measurements to get maximum realistic information. The modern theory of experiments proves that it is not always true that higher number of measurements will give maximum realistic information. Larger the number of measurements, huge will be the total error that enters into the measurement equation. A larger number of measurements led to more costly experimentation. It is necessary to obtain the maximum information while doing a minimum number of experiments. One of the best examples is the Taguchi method. The DOE program is to look which factor effects more on the output of an experiment.

Objectives

As per literary review, we found that the researchers have so far worked on option pricing model using Taguchi's orthogonal array design whereas Taguchi's model can be used in option pricing also. The objectives of this study are:

1. To identify the best level for each parameter.
2. To measures which factors are more important than others.
3. To check whether the value of a call option follows a certain distribution.
4. To measure the percentage contribution of each parameter.

METHODOLOGY

The aim of this paper is to do an experiment on the European call option at one period at three levels. The Taguchi's method (L9 (3^4)) is used to run the trials. In Taguchi's method, only 9 experiments are used instead of 81 as per FFD. The date is taken from the published paper (Dar and Anuradha, 2017). The data gives the complete information to measure the value of the call option by using Black Scholes formula given in Table 1. Finally, the ANOM, S/N ratio, Tukey method is used to look which factor effects more or less and ANOVA gives the percentage contribution of the variables on European call option.

TABLE 1
DATA FOR CALCULATING THE CALL OPTION VALUE BY USING BSM

Levels ↓	Parameters →	S_0	K	r	σ
1		130	100	5%	20%
2		140	105	6%	21%
3		150	110	7%	22%

EUROPEAN CALL OPTION

European call option: "It is a contract that gives rights to the owner, but not obligation to buy the underlying asset (S_0) at a specified price (strike price K) within a specified time (T). It will not exercise before the maturity date. The buyer of the call option believes that the price of an underlying asset goes up in a future date. In this case, the buyer of the call option will decide whether to exercise or not because he is having the rights. At the expiry date T, there are two possibilities a) if the price of the underlying asset S_T is less than strike price K. Then the call option is not exercised and b) if the price of an underlying asset S_T at maturity time is greater than the strike price K, then he will exercise it, i.e. the holder buys the underlying asset at price K and sells it to the market at a price S_T". From above both cases, the payoff at maturity date is:

$$\max(S_T - K, 0)$$

The Black and Scholes developed a formula in order to estimate the values of European call and put option at time $t = 0$ in 1973 (Black and Scholes 1974, Merton 1974, Dar and Anuradha 2017, Hull 2016).

The Black-Scholes formula for European call option without dividend paying is:

$$f(t, S_t) = S_t * N(d_1) - K * e^{-r(T-t)} * N(d_2) \; [1]$$

Where N(*) is the standard cumulative distribution function

$$d_1 = \frac{\ln\left({S_0}/{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

"S_t is the price of an underlying asset at time t, K is the strike price, r is the risk free rate of the interest, $(T-t)$ is the mature time, σ is the volatility of the return of the underlying asset, $N(*)$ cumulative distribution function of the standard normal distribution".

Note **a**) The European style means that the contract will expire only at a set date or maturity date. **b**) In case of call option the underlying asset at mature date must be greater than K. **c**) $S_t * N(d_1)$ is the present value of the underlying asset if the option is exercised and **d**) $K * e^{-r(T-t)}$ is the present value of the strike price K if the option is exercised.

DESIGN OF EXPERIMENT (DOE) USING TAGUCHI ORTHOGONAL ARRAY

Taguchi method is a popular statistical model developed by G. Taguchi. At starting it was used for only improving the quality of products (mainly manufactured goods). Nowadays it is used in every field in order to minimise the number of trials, time, cost and resources. This method is based on the orthogonal array experiments which give a much-reduced variance for the experiment with the optimum setting. Taguchi orthogonal array design is a type of design that is based on a design matrix and it allows you to consider a selected subset of combinations of various factors at different levels. It is balanced and ensures that all levels of all parameters are considered equally. In this study, the four parameters are varied at three levels and on the bases of levels and parameters, the orthogonal array L9 is selected. The procedure for Taguchi method is shown in Figure 1.

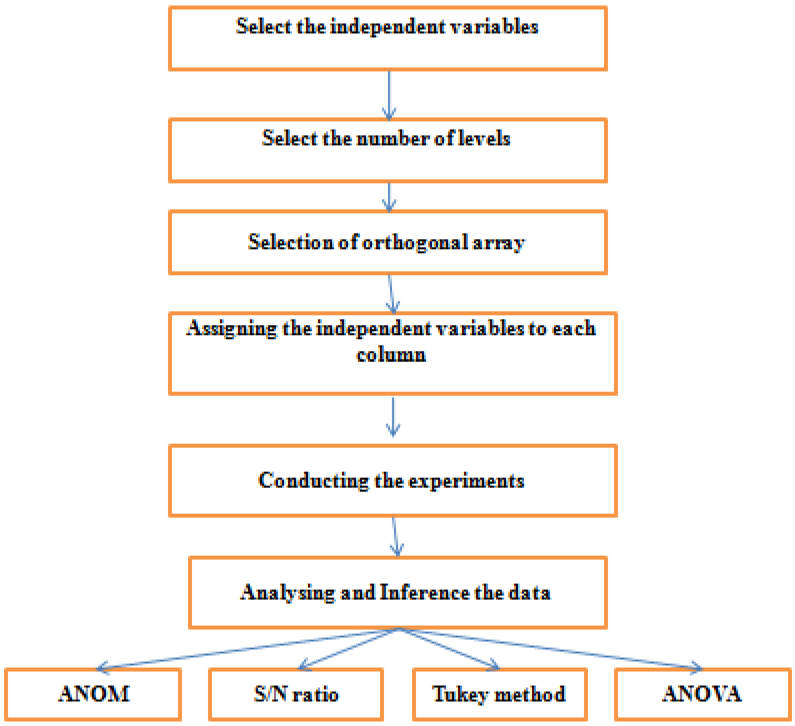

FIGURE 1
PROCEDURE FOR TAGUCHI ORTHOGONAL ARRAY METHOD

The experiment runs with four parameters at three levels are determined by using the Taguchi L9 orthogonal array. The MINITAB software determines 9 trails instead of 81 as per FFD. The four parameters are: underlying asset S_0, strike price K, interest rate r, and volatility σ at three levels are summarised in Table 1.

The experiment layouts for call option process parameters by using Taguchi L9 approach shown in Table 2. These experiments are not randomly selected but it is based on some well-defined procedure or sampling. The Taguchi L9 orthogonal array approach is appropriated for experimentation and the experimental matrix along with result (value of European call option) using equation (1) is shown in Table 2.

TABLE 2
TAGUCHI'S L9(3^4) ORTHOGONAL ARRAY FOR EUROPEAN CALL OPTION USING BSM

Experiment	S_0	K	r	σ	$f(t, S_t)$ Call option value
1	130	100	0.05	0.20	35.44027
2	130	105	0.06	0.21	32.18835
3	130	110	0.07	0.22	29.25158
4	140	100	0.06	0.22	46.18057
5	140	105	0.07	0.20	42.44147
6	140	110	0.05	0.21	36.32111
7	150	100	0.07	0.21	56.86112
8	150	105	0.05	0.22	50.45923
9	150	110	0.06	0.20	46.71759

RESULT, ANALYSIS, AND DISCUSSION

Model Summary

TABLE 3
MODEL SUMMARY

S	R-sq	R-sq(adj)	R-sq(pred)
0.159971	99.98%	99.97%	99.88%

R-sq is a regression coefficient. It is a statistical tool that measures how much the data is close in order to fit the regression line. In this model R-sq value is 99.98% which indicates that the fit of the experimental data is satisfactory. The R-sq is approximately equal to 1 that means the regression line perfectly fits the data as shown in table 3. S is the standard deviation of the data points. It indicates that how far a data fall from the fitted value.
The Minitab software was used in order to model for the call option value, the regression equation is given below:

Regression Equation

$$\text{value} = 28.898 + 9.5263*S_0 - 4.3653*K + 1.0556\ 8*r + 0.2153*\sigma$$

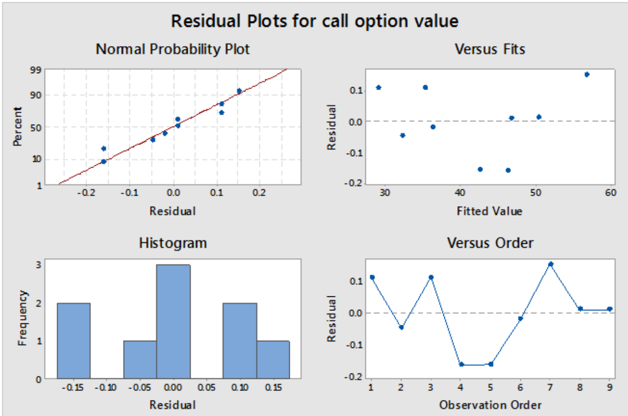

FIGURE 2
RESIDUAL PLOT FOR CALL OPTION VALUE

The following points explain the figure 2.

a. In normal probability plot, there is no outliers exist which means that the values of call options (data) follow a normal distribution and the factors are influencing the response.
b. In versus fits plot (fitted value vs. residual), the plot indicates that the relationship between the data is non-linear and the variance is constant.
c. The Histogram indicates that there is no outlier exists and the data are not skewed.
d. In versus plot (residuals versus observation order), the plot indicates that the data have systematic effects.

Analysis of Mean (ANOM)

The response mean (ANOM) is the average response for each combination of control parameters (factors) levels in a statistic Taguchi method. The aim of this method is to identify which parameter effects more on European call option and also it determines the best combination where the European call option gets maximum value. For each parameter, Minitab software measured the average of the response (call option value) at each level of the parameter. The delta identifies the size of effect by the taking the difference between the highest and the lowest value of average for a parameter and the rank in the response Table 4 helps us to identify which parameter effects more. The parameter with the highest delta value is given rank 1, the parameter with the second highest delta is given rank 2, and so on

TABLE 4
RESPONSE TABLE FOR MEAN (ANOM)

Level	S_0	K	r	σ
1	32.2934	**46.16065**	40.7402	41.53311
2	41.64772	41.69635	41.6955	41.79019
3	**51.34598**	37.43009	**42.85139**	**41.96379**
Range	19.05258	8.73056	2.111187	0.430683
Rank	1	2	3	4

Range=Max-Min.

The selected numbers **(bold)** in Table 4 are the maximum in every column, as per range we set the ranking for all the parameters (higher range=rank 1 and so on). The ANOM gives us an idea about which parameter affect more on option pricing. Table 4 clearly shows that which parameter affects more. The rank indicates that the underlying asset S_0 affects more and the volatility σ affects less on call option. The bolded values are maximum in every column and we also conclude that the best combination is $\mathbf{S_0\,3 * K1 * r3 * \sigma 3}$.

The best combination has been determined $S_0\,3 * K1 * r3 * \sigma 3$ because it will provide us maximum value of call option. In order to prove, we will choose some combinations from Table 2 (experiment no. 3 and 4) and some random combinations are selected. The variations of call options with different combinations are presented in figure 3. It is shown that the optimal combination has the highest value (56.90183) than all four combinations and also we assume that there exist the additively of effects of different parameters (Wang et al. 2016).

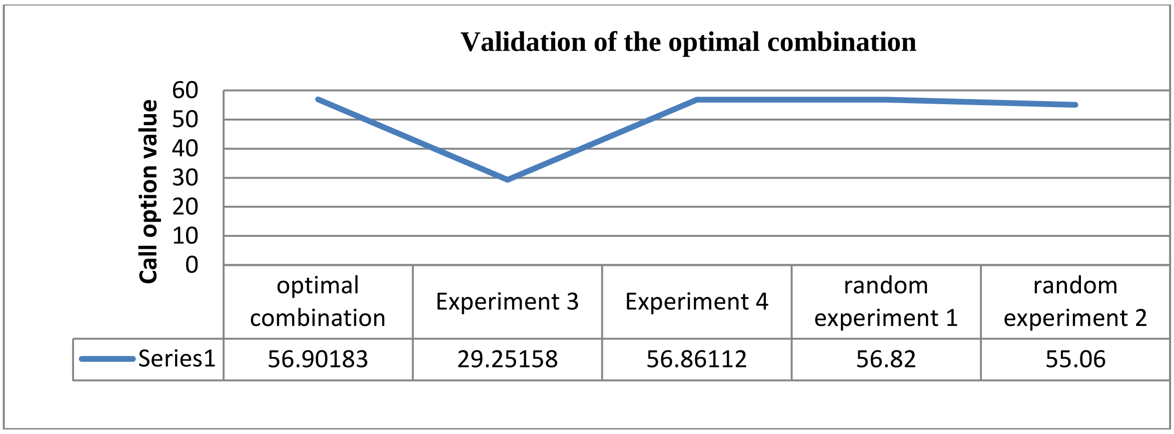

FIGURE 3
VARIATIONS OF EUROPEAN CALL OPTION WITH DIFFERENT COMBINATIONS

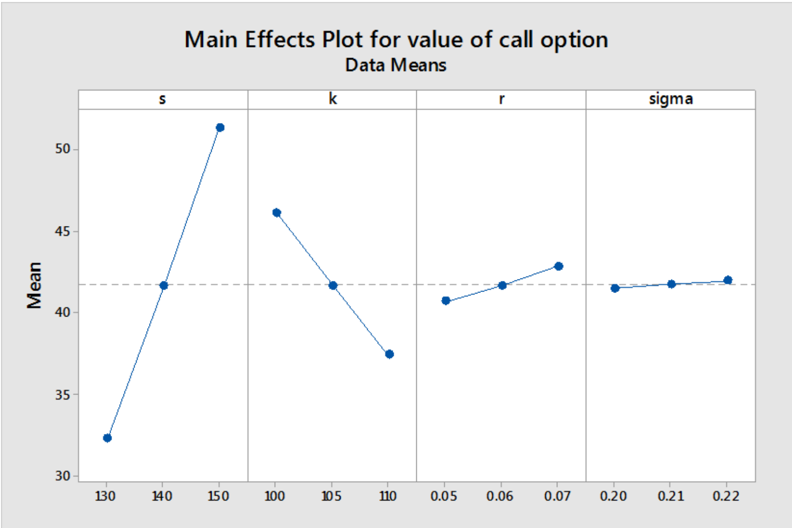

FIGURE 4
MAIN EFFECTS PLOT FOR CALL OPTION

When the lines are parallel to the x-axis, then there is no effect on the response variable. Figure 4 indicates that all the lines are not parallel to the x-axis, which means all the factors effects on the call option. The slope of the underlying asset S_0 *or* S is high, which indicated that it affects more on call option and the slope of sigma σ is less, which indicates it affects less on call the option.

Signal-to-Noise Ratio (Larger-is-Best)

The experiment layout using Taguchi L9 orthogonal array with responses values of European call option using BSM. The S/N ratios of all the four parameters at three levels were calculated and are shown in Table 5.

TABLE 5 RESPONSE TABLE FOR SIGNAL TO NOISE RATIOS LARGER IS BETTER				
Level	S_0	K	r	σ
1	30.16	**33.13**	**32.08**	32.31
2	32.35	32.26	32.28	**32.15**
3	**34.18**	31.31	32.33	32.22
Delta	4.03	1.82	0.24	0.16
Rank	**1**	**2**	**3**	**4**

The underlying asset price S_0 (Delta=4.03, Rank=1) has the largest effect and volatility σ (Delta = 0.16, Rank = 4) has the smallest effect on call option. As per Table 5 the rank indicates that which factor effects more on the European call option. It also shows that the underlying asset affects more and volatility affects less on call option.
In order to measure which factor effects more on response variables? The best method is to compare the slope of lines with relative magnitude.

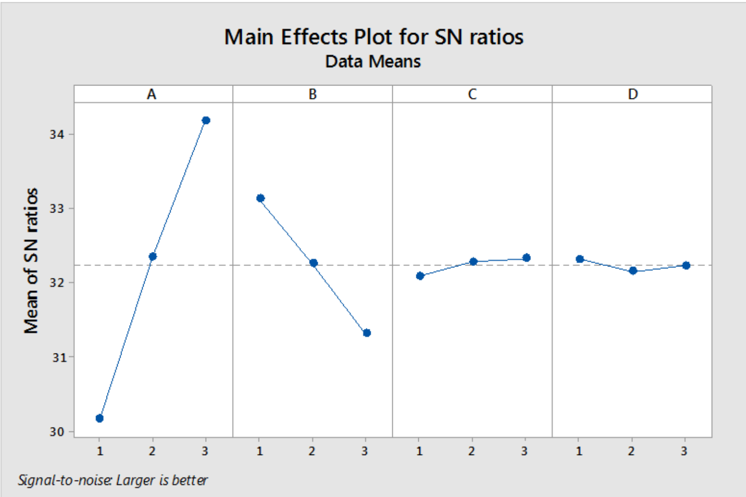

FIGURE 5
MAIN EFFECTS PLOT S/N RATIO

If the line in Figure 5 is horizontal then there is no effect. But all the lines are not horizontally which mean that each factor effects on the call option. It clearly shows that underlying asset S_0 *or* S (A) effect more as compared to others and the volatility σ (D) effects less.

We can use Tukey method also in order to identify which parameter effects more in call option using BSM.

Tukey Pairwise Comparisons

TABLE 6 GROUPING INFORMATION USING THE TUKEY METHOD AND 95% CONFIDENCE					
Factor	N	Mean	Grouping		
S_0	9	140.00	A		
K	9	105.00		B	
σ	9	0.21000			C

r	9	0.06000			C
Note: Means that do not share a letter are significantly different.					

In this result, the Table 6 shows that the group A contains a factor S_0, B contains factor K and C contains σ and r. The result shows that it is not sharing any letter. The groups that shares the letter are not significant different. Factor S_0 and K is not sharing the letter, which indicates that S_0 has a significantly higher mean than factor K, σ, and r.

Analysis of Mean (ANOVA)

In order to measure the percentage contributions of each independent variable in European call option using BSM. An application of ANOVA is used in order to estimate the percentage of each parameter. It shows the relation between the response variable and the predictor variable. In order to estimate the percentage contribution, we need regression model known as ANOVA. It is defined as "sum of squares of a parameter by total sum of squares". Table 7 shows the contribution of each parameter.

The percentage contribution of the parameters that are shown in Table 6 can be calculated as

$$percentage\ contribution = \frac{sum\ of\ square\ of\ a\ parameter}{total\ sum\ of\ squares}$$

TABLE 7
ANOVA

Source	Adj SS	Adj MS	F-Value	P-Value	Percentage contribution	Rank
S	544.501	544.501	21277.24	0.000	81.77%	1
K	114.334	114.334	4467.78	0.000	17.17%	2
r	6.686	6.686	261.25	0.000	1.004%	3
σ/sigma	0.278	0.278	10.87	0.030	0.0424%	4
Error	0.102	0.026				
Total	665.901					
Note: at 95% confidence interval ($\alpha = 0.05$)						

The P-value of all the parameters are less than ($\alpha = 0.05$), because of this we can conclude that there is a statistically significant differ. So, all the factors are effects on call option differently. In order to measure which parameter effects more or less on call option at one period, the ANOVA is used.

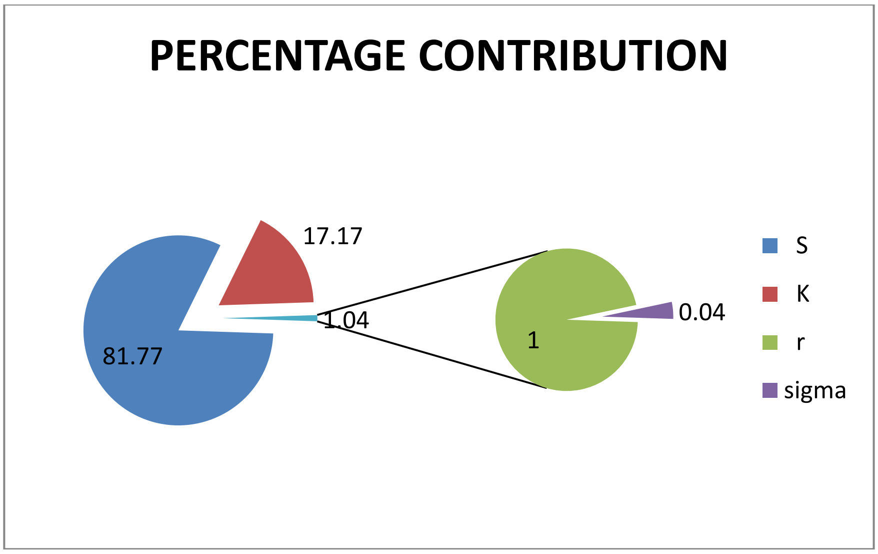

FIGURE 6
PERCENTAGE CONTRIBUTION

The percentage of each parameter is defined as the significance rate of the process parameters on the value of call option. The percentage (%) numbers represent that the underlying asset price at time t=0, the strike price, the interest rate and the volatility have a significant effect on pricing of European call option using BSM. It can be observed in table 9 that the underlying asset price at time$(t = 0)$ S_0, the strike price K, the interest rate r and volatility σ effects the call option by 81.77%, 17.17%, 1.004% and 0.0424% respectively are shown in Table 7 and Figure 6.

Assumption: The percentage contribution will vary with the change in data set.

CONCLUSION

This study discussed an application of the Taguchi L9 orthogonal array. It is based on the European call option using BSM at one period. The four parameters: the underlying asset S, the strike price K, the interest rate r and the volatility σ on European call option at three levels are used. In general 3^4=91 trials were supported to be conducted. However, only 9 trials were done.The conclusion of this study is summarised below:

a. The values of a call option follow a normal distribution because the values approximately in a straight line and there are no outlier exist.
b. The ANOM is being used in order to identify the best level for every four parameters. The best combination in this study is $\boldsymbol{S_0 3 * K1 * r3 * \sigma 3}$**.** This combination gives the maximum value of European call option as compared to all other possible combinations.
c. The ANOM and S/N ratio is being used in order to identify which parameter effects more or less on European call option. The rank showed in Table 6, 7 and 8 that underlying asset S_0 impact more and volatility σ impact/effect less on call option.
d. The Tukey method also used in order to identify the underlying asset S_0 has a significantly higher mean than another factor.
e. The percentage contribution of the underlying asset price S_0 at time $t = 0$, the strike price K, the interest rate r and volatility σ effects the call option by 81.77%, 17.17%, 1.004% and 0.0424% respectively.

In this paper, the Taguchi L9 orthogonal array was successfully applied in order to identify which parameter effects more on European call option using Black Scholes Model.

Table 8 Ranks			
Rank = 1 Underlying asset S	Rank = 2 The strike price K	Rank = 3 The interest rate r	Rank = 4 The volatility σ

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