TWO WARE-HOUSES FUZZY INVENTORY MODEL FOR DETERIORATING ITEMS WITH RAMP TYPE DEMAND AND SHORTAGES

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ABSTRACT

In this paper we developed a fuzzy inventory model for single spoilage two-parameter weibull-distribution degradation rate, ramp type demand, and partial backordering at a constant rate. In the current market scenario, an increase in the cost of the inverter affecting the total cost of inventory costs due to inflation can increase at any time of the order length.The increase in the cost of the components of the inventory cannot be pre-determined due to the uncertainty of the market situation. Therefore, we have considered the interval based fuzzy concept to handle the uncertainty condition. Ordering cost, the cost of holding in both ware-houses is considered a triangular fuzzy number.

Keywords: Weibull deterioration distribution; Partial backlogging; Rramp type demand; Fuzzy holding cost; Ordering cost.

INTRODUCTION

In traditional models, many researchers have considered the rate of demand to be constant, linear time dependent, stock dependent or accelerating over time and the same trend of considering this type of demand still continues but it is not always true that demand occurs in the same pattern. The assumption of a constant demand rate is generally valid in the mature phase of a product's life cycle. Several models have developed inventory models for items stored in two ware-houses under modelling assumptions. The rate of demand in these models is assumed to be constant over time. However, in practice a person will accept the demand for separation over time.Most classical inventory models assumed that the utility of the inventory remains constant during the period of their storage. But in real life, degradation occurs during the storage period. The deterioration of physical objects is a common phenomenon in the real world that can occur for various reasons and has attracted a lot of attention from various researchers.In recent years, the problem of inventory worsening has received considerable attention. Most products such as medicine, blood, fish, alcohol, gasoline, vegetables, and radioactive chemicals have self-life, and once spoiled they begin to deteriorate. The listed researchers, above, have taken care of deteriorating objects in their models and have developed models accordingly. In addition to inventory deterioration, limited storage is also a major practical problem for real life.Due to lack of large storage space at important market places, one should be forced to own a small ware-

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house at important market places. At times, it may be profitable for the retailer to order quantities in excess of its own warehouse capacity. In this situation retailers require additional space to store bulk purchased items and therefore prefer to rent the house for a limited period. A specially equipped storage facility is required to reduce the amount of spoilage. The cost of building such a storage facility for a limited period is usually excessive. Therefore, it may be difficult for a retailer to have such a storage facility of its own at retail outlets. To handle this situation, another storage space is required, making the necessary facilities available. To launch some new products e.g. Fashionable items, clothing, electronic items, mobiles etc. The uncertainty of change in any component of an inventory model can occur and cannot be determined in advance until we arrive at that position or time.For example increasing prices, affecting total inventory costs or demand, shortages, etc. It is not easy to estimate how much? and/or when increases / decreases in components of an inventory model occur for the foreseeable future? One of management's most concerns is to decide when and how much to order so that the costs associated with the inventory system are minimized.This is more important when some or more products in the inventory are deteriorating. These types of situations can be dealt with the help of fuzzy based concepts such as fuzzy set theory, fuzzy numbers, etc., which were introduced by Zadeh (1965) in their paper, followed by many using fuzzy set theory in fuzzy environments. Papers have been developed. Zaidh and Bellman (1970) considered an inventory model when making decisions in fuzzy environments. Jain (1976) developed a fuzzy inventory model when making decisions in the presence of fuzzy variables. DuBois and Prade (1978) defined some operations on fuzzy numbers. In general, demand is assumed to be either constant or increasing over time (Kumar et al., 2007). An economic production volume is developed with fuzzy demand and rate of decline. Syed and Aziz (2007) consider a signed distance method for a fuzzy inventory model without drawbacks. De and Rawat (2001) developed a fuzzy inventory model without reduction using triangular fuzzy number. Jaggi et al. (2012) developed a fuzzy inventory model for deteriorating goods over time demand and scarcity. Datta and Kumar (2012) considered an optimal replenishment policy for an inventory model without reduction by assuming fajita in demand. Halim et al. (2008) developed a fuzzy inventory model for perishable goods with stochastic demand, partial backlogging, and fuzzy deterioration rates. Gani and Maheshwari (2010) discussed the retailer's order policy under two levels of payment delays and considered the demand and selling price as a triangular fuzzy number. He used the Graded Mean Integration Representation Method for the differentiation. Halim et al. (2010) addressed a very sizing problem with stochastic machine breakdown and fuzzy repair time using the signed distance method in an unreliable production system.Singh and Singh (2008) developed a fuzzy inventory model for the finite rate of replenishment using the signed distance method. Jaggie et al. (2018a) discussed a decrease in trade with inventory quality deteriorating commodities and partially backlogged short lived shortages for thoughtful decision making. Jaggi et al. (2018b) discussed the tau-warehouse inventory model for non-instantaneous perishable items under various dispatch policies. Khanna et al. (2017) detailed about inventory modelling for incomplete quality items with price dependent demand and short backorder sales under credit financing. Jaggi et al. (2018c) differentially defined criteria for poor quality items with increasing demand and partial backlogging. Khanna et al. (2016) defined credit for deterioration of goods of imperfect quality with acceptable quality. Jaggi et al. (2015) discussed an inventory model for deteriorating goods with ramp type demand under trapped environment. Yadav et al. (2020) studied supply chain management and its impact of industrial development on the defined electronic components warehouse. Yadav and Swamy (2018a;b; 2019a;b) discussed about an

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inventory model for non-instantaneously deteriorating commodities with variable holding costs under two-storage and a volume flexible two-warehouse model with volume fluctuations and Holding costs with inflation and a partial backlog production. Inventory lot-size model with time-varying hold and time-varying cost and Weibull decline and supply chain model for items deteriorating with linear stock dependent demand under the Impression and Inflationary Environment (Pandey et al., 2019; Yadav et al., 2017a;b;c) It is discussed that with the chain inventory model, warehouse and distribution centers for perishable goods and supply centers for the deteriorating goods and warehouses for the chemical industry supply chain, With distribution centers using the artificial BE colony algorithm. Influence of inflation on the two-warehouse inventory model for commodities deteriorating over time and demand and two storage systems for two warehouses with soft computing optimization and an inflationary inventory model for deterioration of items under the supply chain inventory model.

In literature, own warehouses are abbreviated as OWs, and rented warehouses as RWs, and it is generally assumed that rent warehouses compare to own warehouses Has better storage facilities available and because of this, the rate decline is smaller than OW resulting in higher holding costs on RW, so retailers prefer to consume goods from RW and then OW first.Inspired by the above papers, we developed a two-ware-house inventory model for the demand rate as a general ramp-type function of time and studied the effect of the two-parameter Weibull distribution degradation rate and fajita with fuzzy triangular numbers.Both the ordering cost and the holding costin ware-house are treated as fuzzy triangular numbers and the signed distance method is used to defreeze the total inventory cost. Only one item is included in this study.Shortage is allowed and partially backordered at a constant rate and the results of crisp and fuzzy models are compared with the help of numerical examples. Sensitivity is also played for both model son limiting parameters.

ASSUMPTION AND NOTATIONS

Assumptions

The following assumptions were used in this research (Table 1)

1	Demand rate is ramp type.
2	The lead time is zero or negligible and initial inventory level is zero.
3	The replenishment rate is infinite.
4	Shortages are allowed and partially backordered at constant rate.
5	The holding cost is constant and higher in RW than OW.
6	The deteriorated units cannot be repaired or replaced during the period under review.
7	Deterioration occurs as soon as items are received into inventory.
8	Deterioration rate is time dependent and follows a two parameter Weibull distribution Wheredenote scale parameter and β, h>1denote the shape parameter.

Notation

The following notation is used throughout the paper:

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Demand rate= $f\left(t_i\right)=\begin{bmatrix} f\left(u\right) & \text{if } t_i>u \text{ for } i=1,2,3 \\ f\left(t_i\right) & \text{if } t_i \le u \end{bmatrix}$

W =Capacity of OW

α =Scale parameter of the deterioration rate in OW and 0<α<1

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g=Scale parameter of the deterioration rate in RW,α> g

h=Shape parameter of the deterioration rate in RW and h>1.

β =Shape parameter of the deterioration rate in OW and β>1.

f_d =Fraction of the demand backordered during the stock out period

C_0 =Ordering cost per order

D_c =Deterioration cost per unit of deteriorated item in RW and OW

H_w =Holding cost per unit per unit time in OW

H_r =Holding cost per unit per unit time in RWsuch that $\left(h_r-h_o\right)>0$

S_c =Backlogging cost per unit per unit time

L_c =Opportunity cost for lost sales per unit

$Q_{max,i}$ =Maximum order quantity at the end of cycle length for i=1,2,3

T_1 =Time with positive inventory in RW

T_1+T_2 =Time with positive inventory in OW

T_3 =Time when shortage occurs in OW

T=Length of the cycle i.e. $T=T_1+T_2+T_3$

$I_i j^t(t_i)$=Inventorylevelforthesystemattimetisuchthat0?ti?Tiandforcasesi=1,2,3

R_{ti} =Inventory level in RW at time t_i=0 for cases i= 1, 2, 3

φ^{ti} =Inventory cost per cycle for cases i=1,2,3

φ^t =Optimal inventory cost per cycle per unit of time

The rate of deterioration is given as follows:

t_i =Time to deterioration, $t_i>0$

Instantaneous rate of deterioration in OW

$\theta_2\left(t_i\right)=g\, h t_i^{(h-1)}$ where 0<g<1 and h>1

{~ Sign represent the fuzziness of the parameters}

MATHEMATICAL MODEL

Crisp Model

Case-1: $0\le u \le T_1$

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Therefore the inventory level is governed by the following differential equations

$$\frac{dI_{11}^{t}(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} I_{11}^{t}(t_1) - f(t_1) \qquad 0 \le t_1 \le u \qquad (1.1)$$

$$\frac{dI_{12}^{t}(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} I_{12}^{t}(t_1) - f(u) \qquad u \le t_1 \le T_1 \qquad (1.2)$$

$I_{11}^{t}(u) = I_{12}^{t}(u)$ and $I_{12}^{t}(T_1) = 0$ the solution of 1.1 & 1.2 are resp.

$$I_{11}^{t}(t_1) = \left(f(u)\left\{ (T_1 - u) + \frac{\alpha}{\beta+1}\left(T_1^{\beta+1} - u^{\beta+1}\right) \right\} - \int_{t_1}^{u} f(x) e^{\alpha x^{\beta}} dx \right) e^{-\alpha t_1^{\beta}} \qquad (1.3)$$

$$I_{12}^{t}(t_1) = \left(f(u)\left\{ (T_1 - u) + \frac{\alpha}{\beta+1}\left(T_1^{\beta+1} - u^{\beta+1}\right) \right\} \right) e^{-\alpha t_1^{\beta}} \qquad (1.4)$$

$$\text{RW is } I_{1}^{t}(0) = (R_{t1} - w)$$

$$R_{t1} = w + \left(f(u)\left\{ (T_1 - u) + \frac{\alpha}{\beta+1}\left(T_1^{\beta+1} - u^{\beta+1}\right) \right\} - \int_{0}^{u} f(x) e^{\alpha x^{\beta}} dx \right) \qquad (1.5)$$

Inventory level time interval (0 T_1) in OW due to malfunction only until the inventory reaches zero on RW.

$$\frac{dI_{13}^{t}(t_1)}{dt_1} = -ght_1^{h-1} I_{13}^{t}(t_1) \qquad 0 \le t_1 \le T_1 \qquad (1.6)$$

$$\frac{dI_{14}^{t}(t_2)}{dt_2} = -ght_1^{h-1} I_{14}^{t}(t_2) - f(t_2) \qquad 0 \le t_2 \le T_2 \qquad (1.7)$$

$I_{3}^{t}(0) = w$ and $I_{4}^{t}(T_2) = 0$. The key of (1.6) & (1.7) are resp.

$$I_{13}^{t}(t_1) = we^{-gt_1^{h}} \qquad (1.8)$$

$$I_{14}^{t}(t_2) = \left(f(u)\left\{ (T_1 - t_2) + \frac{g}{h+1}\left(T_1^{\beta+1} - t_2^{\beta+1}\right) \right\} \right) e^{-gt_2^{h}} \qquad (1.9)$$

Both warehouses are empty during the time interval (0 T_3) at $T_3 = 0$.

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$$\frac{dI_{15}^{t}(t_3)}{dt_3} = -f_d\, f(u) \qquad 0 \le t_3 \le T_3$$

(1.10)

$I_{15}^{t}(0) = 0$ the key of equation (1.10) is given as

$$I_{15}^{t}(t_3) = -f_d\, f(u)\, t_3 \tag{1.11}$$

The lost sales volume of inventory throughout the shortage time is

$$LS = (1\text{-}f_d)f(u)T_3$$

$$\int_0^u f(t_1)\,dt_1 + \int_u^{t_1} f(u)\,dt_1$$

Therefore the amount of inventory that has been spoiled during this period is

$$D_R = R_{t1} - \int_0^u f(t_1)\,dt_1 - \int_u^{t_1} f(u)\,dt_1$$

$$T_1 + T_2 \text{at OW is } \int_0^{T_2} f(u)d\, t_2$$

$$D_w = W - \int_0^{T_2} f(u)\,dt_2$$

The total applicable inventory cost per unit of time

$$\varphi^{t1}(T_1, T_2, T_3) = \frac{1}{T}\begin{bmatrix} C_o + H_r\left(\int_0^u I_{11}^{t}(t_1)\,dt_1 + \int_0^{T_1} I_{12}^{t}(t_1)\,dt_1\right) + \\ H_w\left(\int_0^{T_1} I_{13}^{t}(t_2)\,dt_2 + \int_0^{T_2} I_{14}^{t}(t_2)\,dt_2\right) + \\ s_c\left(\int_0^{T_3}\left(-I_{15}^{t}(t_3)\right)dt_3\right) + L_c LS + D_c(D_{1R} + D_{1w}) \end{bmatrix} \tag{1.12}$$

Case-2: $0 \le u \le T_2$

$$\frac{dI_{21}^{t}(t_1)}{dt_1} = -\alpha\beta t_1^{\beta-1} I_{21}^{t}(t_1) - f(t_1) \qquad 0 \le t_1 \le T_1 \tag{2.1}$$

$I_{21}^{t}(T_1) = 0$. The solution of equation 2.1 is

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$$\mathrm{I}_{21}^{t}(t_1)=\left(\int_0^{T_1} f(x)e^{\alpha x^{\beta}}dx-\int_0^{t_1} f(x)e^{\alpha x^{\beta}}dx\right)e^{-\alpha t_1^{\beta}} \tag{2.2}$$

RW is $I_2 1^t(0)=(R_{t2}-W)$ and $T_1 > 0$

$$R_{t2}=\left(W+\int_0^{T_1} f(x)e^{\alpha x^{\beta}}dx)\right)>W$$

Applying the same logic as OW to the time intervals (0 T_1) and (0 T_2), the following differential equations are obtained

$$\frac{d\mathrm{I}_{22}^{t}(t_1)}{dt_1}=-ght_1^{h-1}\mathrm{I}_{22}^{t}(t_1) \qquad 0\le t_1\le T_1 \tag{2.3}$$

$$\frac{d\mathrm{I}_{23}^{t}(t_2)}{dt_2}=-ght_2^{h-1}\mathrm{I}_{23}^{t}(t_2)-f(t_2) \qquad 0\le t_2\le u \tag{2.4}$$

$$\frac{d\mathrm{I}_{24}^{t}(t_2)}{dt_2}=-ght_2^{h-1}\mathrm{I}_{24}^{t}(t_2)-f(u) \qquad u\le t_2\le T_2 \tag{2.5}$$

$\mathrm{I}_{22}^{t}(0)=\mathrm{W}$ and $\mathrm{I}_{24}^{t}(T_2)=0$ the solution of (2.3), (2.4) & (2.5 are respectively

$$\mathrm{I}_{22}^{t}(t_1)=\mathrm{w}e^{-gt_1^{h}} \tag{2.6}$$

$$\mathrm{I}_{23}^{t}(t_2)=\left(We^{-gT_1^{h}}-\int_0^{t_2} f(x)e^{gx^{h}}dx\right)e^{-gt_2^{h}} \tag{2.7}$$

$$\mathrm{I}_{24}^{t}(t_2)=\left(f(u)\left\{(T_2-t_2)+\frac{g}{h+1}\left(T_2^{\beta+1}-t_2^{\beta+1}\right)\right\}\right)e^{-gt_2^{h}} \tag{2.8}$$

Furthermore, both warehouses are empty during the time interval (0 T_3) at T_3 = 0, and a portion of the shortfall is returned to the next “replenishment” and is similar to Equation (1.10) of Case-1

$$\mathrm{I}_{15}^{t}(t_3)=-f_d\, f(u)\,t_3 \tag{2.9}$$

The lost sales volume of inventory throughout the shortage period is

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$$LS=(1-f_d)f(u)T_3$$

RW is $\int_0^{T_1} f(t_1)dt_1$ and therefore

RW is the amount of inventory deteriorating

$$D_{2R}=R_{t2}-\int_0^{T_1} f(t_1)dt_1$$

The total demand during time epoch T_1+T_2 at OW is $\int_0^{u} f(t_2)dt_2-\int_u^{T_2} f(u)dt_2$ and amount of inventory deteriorated during the period T_1+T_2 at OW is

$$D_{2w}=W-\int_0^{u} f(t_2)dt_2-\int_u^{T_2} f(u)dt_2$$

$$\varphi^{t2}(T_1,T_2,T_3)=\frac{1}{T}\begin{bmatrix} C_o+h_R\left(\int_0^{T_1} I_{21}^t(t_1)dt_1\right)+ \\ h_w\left(\int_0^{T_1} I_{22}^t(t_2)dt_2+\int_0^{u} I_{23}^t(t_2)dt_2+\int_u^{T_2} I_{24}^t(t_2)dt_2\right)+ \\ s_c\left(\int_0^{T_3}(-I_5^t(t_3))dt_3\right)+L_cLS+D_c(D_{2R}+D_{2w}) \end{bmatrix} \quad (2.10)$$

Case-3: $0\leq u\leq T_3$

$$\frac{dI_{31}^t(t_1)}{dt_1}=-\alpha\beta t_1^{\beta-1}I_{31}^t(t_1)-f(t_1) \qquad 0\leq t_1\leq T_1 \qquad (3.1)$$

$I_{21}^t(T_1)=0$. The answer of (3.1) is

$$I_{31}^t(t_1)=\left(\int_0^{T_1} f(x)e^{\alpha x^{\beta}}dx-\int_0^{t_1} f(x)e^{\alpha x^{\beta}}dx\right)e^{-\alpha t_1^{\beta}}$$

RW is $I_{31}^t(0)=(R_{t3}-W)$ and $T_1>0$.

$$R_{t3}=\left(W+\int_0^{T_1} f(x)e^{\alpha x^{\beta}}dx)\right)>W$$

$$\frac{dI_{32}^t(t_1)}{dt_1}=-ght_1^{h-1}I_{32}^t(t_1) \qquad 0\leq t_1\leq T_1 \qquad (3.2)$$

$$\frac{dI_{33}^t(t_2)}{dt_2}=-ght_2^{h-1}I_{33}^t(t_2)-f(t_2) \qquad 0\leq t_2\leq T_2 \qquad (3.3)$$

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With boundary conditions $I^t_{32}(0) = W$, and $I^t_{33}(T_2) = 0$,the solution of (3.2)& (3.3) are respectively

$$I^t_{32}(t_1) = we^{-gt_1^h} \tag{3.4}$$

$$I^t_{33}(t_2) = \left(\int_0^{T_1} f(x)e^{gx^h}dx - \int_0^{t_1} f(x)e^{gx^h}dx\right)e^{-gt_1^h} \tag{3.5}$$

Furthermore, both warehouses are empty during the time interval (0 T_3) at $T_3 = 0$,

$$\frac{dI^t_{34}(t_3)}{dt_3} = -f_d f(t_3) \qquad 0 \le t_3 \le u \tag{3.6}$$

$$\frac{dI^t_{35}(t_3)}{dt_3} = -f_d f(u) \qquad 0 \le t_3 \le T_2 \tag{3.7}$$

With boundary conditions $I^t_{34}(0) = 0$, and $I^t_{34}(u) = I^t_{35}(u)$,the answer of (3.6) & (3.7) are respectively

$$I^t_{34}(t_3) = -f_d \int_0^{t_3} f(x)dx \tag{3.8}$$

$$I^t_{35}(t_3) = -f_d\left(f(u)(t_3 - u) + \int_0^u f(x)dx\right) \tag{3.9}$$

The lost sales volume of inventory during the shortage period is

$$LS = (1 - f_d)\left(\int_0^u f(t_3)dt_3 + \int_0^{T_3} f(t_3)dt_3\right)$$

RW is $\int_0^{T_1} f(t_1)dt_1$ and therefore

RW is the amount of inventory deteriorating

$$D_{3R} = R_{t3} - \int_0^{T_1} f(t_1)dt_1$$

$$T_1 + T_2 \text{at OW is} \int_u^{T_2} f(t_2)dt_2$$

$$D_{3w} = W - \int_u^{T_2} f(t_2)dt_2$$

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$$\varphi^{t3}(T_1,T_2,T_3)=\frac{1}{T}\begin{bmatrix} C_o+h_R\left(\int_0^{T_1} I_{31}^{t}(t_1)\,dt_1\right)+ \\ h_w\left(\int_0^{T_1} I_{32}^{t}(t_2)\,dt_2+\int_0^{T_2} I_{33}^{t}(t_2)\,dt_2\right)+ \\ s_c\left(\int_0^{u}-I_{34}^{t}(t_3)\,dt_3+\int_u^{T_3}-I_{35}^{t}(t_3)\,dt_3\right)+L_cLS+D_c(D_{3R}+D_{3w}) \end{bmatrix} \quad (3.10)$$

Fuzzy Model

The model developed above is a crisp mock-up in which all criteria are set but this is not always possible. In the current global market, the value of parameters such as cost, demand can fluctuate due to inflation and uncertainty of any reason (eg low production, natural hazards, etc.) and it can fluctuate around its own value. . The fluctuations at any time cannot be predetermined until we reach the state of that time.Therefore, the only possibility is to consider the possible range of fluctuations. To deal with this type of uncertain condition, we developed a fuzzy model looking at the ambiguity of some parameters affecting the total inventory cost. We have taken the order cost and the investment cost as fuzzy numbers in both warehouses represented by a triangular number.The model is solved using the signed distance method to reduce the total inventory cost. The results from three different cases are compared with the value of the crisp model. Fazimodel for three different cases is as follows:

Case-1

Using equation (1.12) and fuzzy parameters$C_o = (C_{\tilde{o}1}, C_{\tilde{o}2}, C_{\tilde{o}3})$,$H_r = (H_{\tilde{r}1}, H_{\tilde{r}2}, H_{\tilde{r}3})$,$H_w = (H_{\tilde{w}1}, H_{\tilde{w}2}, H_{\tilde{w}3})$ we have,

$$\tilde{\varphi}^{t1}(T_1,T_2,T_3)=(\tilde{\varphi}_1^{t1}(T_1,T_2,T_3), \tilde{\varphi}_2^{t1}(T_1,T_2,T_3), \tilde{\varphi}_3^{t1}(T_1,T_2,T_3))$$

Where,

$$\tilde{\varphi}_1^{t1}(T_1,T_2,T_3)=\frac{1}{T}\Big[C_{\tilde{o}1}+H_{\tilde{r}1}\left(\int_0^{u} I_{11}^{t}(t_1)dt_1+\int_0^{T_1} I_{12}^{t}(t_1)dt_1\right)+H_{\tilde{w}1}\left(\int_0^{T_1} I_{13}^{t}(t_2)dt_2+ \int_0^{T_2} I_{14}^{t}(t_2)dt_2\right)+ s_c\left(\int_0^{T_3}(-I_{15}^{t}(t_3))dt_3\right)+L_cLS + d(D_{1R}+D_{1w})\Big]$$

$$\tilde{\varphi}_2^{t1}(T_1,T_2,T_3)=\frac{1}{T}\Big[fC_{\tilde{o}2}+fH_{\tilde{r}2}\left(\int_0^{u} I_{11}^{t}(t_1)dt_1+\int_0^{T_1} I_{12}^{t}(t_1)dt_1\right)+fH_{\tilde{w}2}\left(\int_0^{T_1} I_{13}^{t}(t_2)dt_2+ \int_0^{T_2} I_{14}^{t}(t_2)dt_2\right)+ s_c\left(\int_0^{T_3}(-I_{15}^{t}(t_3))dt_3\right)+L_cLS + d(D_{1R}+D_{1w})\Big]$$

$$f\tilde{\varphi}_3^{t1}(T_1,T_2,T_3)=\frac{1}{T}\Big[fC_{\tilde{o}3}+fH_{\tilde{r}3}\left(\int_0^{u} I_{11}^{t}(t_1)dt_1+\int_0^{T_1} I_{12}^{t}(t_1)dt_1\right)+fH_{\tilde{w}3}\left(\int_0^{T_1} I_{13}^{t}(t_2)dt_2+ \int_0^{T_2} I_{14}^{t}(t_2)dt_2\right)+ s_c\left(\int_0^{T_3}(-I_{15}^{t}(t_3))dt_3\right)+L_cLS + d(D_{1R}+D_{1w})\Big]$$

Thus with above fuzzy parameters the total inventory cost in case-1 is given by the eq.

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$$\tilde{\varphi}^{t1}(T_1,T_2,T_3) = \frac{1}{T}\Big[C_{\tilde{o}} + H_{\tilde{r}}\Big(\int_0^u I_{11}^t(t_1)dt_1 + \int_0^{T_1} I_{12}^t(t_1)dt_1\Big) + H_{\tilde{w}}\Big(\int_0^{T_1} I_{13}^t(t_2)dt_2 + \int_0^{T_2} I_{14}^t(t_2)dt_2\Big) + s_c\Big(\int_0^{T_3}\Big(-I_{15}^t(t_3)\Big)dt_3\Big) + L_c LS + d(D_{1R} + D_{1w})\Big] \quad (3.13)$$

Similarly for other two cases using above defined fuzzy parameters the total inventory cost is given by following equations

Case-2

$$f\tilde{\varphi}^{t2}(T_1,T_2,T_3) = \frac{1}{T}\Big[fC_{\tilde{o}} + fH_{\tilde{r}}\Big(\int_0^{T_1} I_{21}^t(t_1)dt_1\Big) + fH_{\tilde{w}}\Big(\int_0^{T_1} I_{22}^t(t_2)dt_2 + \int_u^{T_2} I_{24}^t(t_2)dt_2\Big) + s_c\Big(\int_0^{T_3}(-I_5^t(t_3))dt_3\Big) + L_c LS + D_c(D_{2R} + D_{2w})\Big] \quad (3.14)$$

Where,

$$f\tilde{\varphi}_1^{t2}(T_1,T_2,T_3) = \frac{1}{T}\Big[fC_{\tilde{o}1} + fH_{\tilde{r}1}\Big(\int_0^{T_1} I_{21}^t(t_1)dt_1\Big) + fH_{\tilde{w}1}\Big(\int_0^{T_1} I_{22}^t(t_2)dt_2 + \int_u^{T_2} I_{24}^t(t_2)dt_2\Big) + s_c\Big(\int_0^{T_3}(-I_5^t(t_3))dt_3\Big) + L_c LS + D_c(D_{2R} + D_{2w})\Big]$$

$$f\tilde{\varphi}_2^{t2}(T_1,T_2,T_3) = \frac{1}{T}\Big[fC_{\tilde{o}2} + fH_{\tilde{r}2}\Big(\int_0^{T_1} I_{21}^t(t_1)dt_1\Big) + fH_{\tilde{w}2}\Big(\int_0^{T_1} I_{22}^t(t_2)dt_2 + \int_u^{T_2} I_{24}^t(t_2)dt_2\Big) + s_c\Big(\int_0^{T_3}(-I_5^t(t_3))dt_3\Big) + L_c LS + D_c(D_{2R} + D_{2w})\Big]$$

$$f\tilde{\varphi}_3^{t2}(T_1,T_2,T_3) = \frac{1}{T}\Big[fC_{\tilde{o}3} + fH_{\tilde{r}3}\Big(\int_0^{T_1} I_{21}^t(t_1)dt_1\Big) + fH_{\tilde{w}3}\Big(\int_0^{T_1} I_{22}^t(t_2)dt_2 + \int_u^{T_2} I_{24}^t(t_2)dt_2\Big) + s_c\Big(\int_0^{T_3}(-I_5^t(t_3))dt_3\Big) + L_c LS + D_c(D_{2R} + D_{2w})\Big]$$

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Case-3

$$f\tilde{\varphi}^{t3}(T_1,T_2,T_3) = \frac{1}{T}\Big[f\mathrm{C}_{\tilde{o}} + fH_{\tilde{r}}\Big(\int_0^{T_1}\mathrm{I}_{21}^{\mathrm{t}}(\mathrm{t}_1)d\mathrm{t}_1\Big) + fH_{\tilde{w}}\Big(\int_0^{T_1}\mathrm{I}_{22}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2 + \int_u^{T_2}\mathrm{I}_{24}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2\Big) + s_c\Big(\int_0^{T_3}(-\mathrm{I}_5^{\mathrm{t}}(\mathrm{t}_3))d\mathrm{t}_3\Big) + L_cLS + \mathrm{D}_c(\mathrm{D}_{2R} + \mathrm{D}_{2w})\Big] \quad (3.15)$$

Where,

$$f\tilde{\varphi}_1^{t3}(T_1,T_2,T_3) = \frac{1}{T}\Big[f\mathrm{C}_{\tilde{o}1} + fH_{\tilde{r}1}\Big(\int_0^{T_1}\mathrm{I}_{21}^{\mathrm{t}}(\mathrm{t}_1)d\mathrm{t}_1\Big) + fH_{\tilde{w}1}\Big(\int_0^{T_1}\mathrm{I}_{22}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2 + \int_u^{T_2}\mathrm{I}_{24}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2\Big) + s_c\Big(\int_0^{T_3}(-\mathrm{I}_5^{\mathrm{t}}(\mathrm{t}_3))d\mathrm{t}_3\Big) + L_cLS + \mathrm{D}_c(\mathrm{D}_{2R} + \mathrm{D}_{2w})\Big]$$

$$f\tilde{\varphi}_2^{t3}(T_1,T_2,T_3) = \frac{1}{T}\Big[f\mathrm{C}_{\tilde{o}2} + fH_{\tilde{r}2}\Big(\int_0^{T_1}\mathrm{I}_{21}^{\mathrm{t}}(\mathrm{t}_1)d\mathrm{t}_1\Big) + fH_{\tilde{w}2}\Big(\int_0^{T_1}\mathrm{I}_{22}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2 + \int_u^{T_2}\mathrm{I}_{24}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2\Big) + s_c\Big(\int_0^{T_3}(-\mathrm{I}_5^{\mathrm{t}}(\mathrm{t}_3))d\mathrm{t}_3\Big) + L_cLS + \mathrm{D}_c(\mathrm{D}_{2R} + \mathrm{D}_{2w})\Big]$$

$$f\tilde{\varphi}_3^{t3}(T_1,T_2,T_3) = \frac{1}{T}\Big[f\mathrm{C}_{\tilde{o}3} + fH_{\tilde{r}3}\Big(\int_0^{T_1}\mathrm{I}_{21}^{\mathrm{t}}(\mathrm{t}_1)d\mathrm{t}_1\Big) + fH_{\tilde{w}3}\Big(\int_0^{T_1}\mathrm{I}_{22}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2 + \int_u^{T_2}\mathrm{I}_{24}^{\mathrm{t}}(\mathrm{t}_2)d\mathrm{t}_2\Big) + s_c\Big(\int_0^{T_3}(-\mathrm{I}_5^{\mathrm{t}}(\mathrm{t}_3))d\mathrm{t}_3\Big) + L_cLS + \mathrm{D}_c(\mathrm{D}_{2R} + \mathrm{D}_{2w})\Big]$$

SENSITIVITY ANALYSIS

Sensitivity analysis is performed on parameter values of example-1.

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Crisp Model

From Table 1 and Table 12, we see that serially the thin millthere value of the total pertinent inventory cost in one suitable unit for Case-1 compared to the other two cases in the model compared to the crisp model (more details refer to Appendix).

TABLE 1
CRISP MODEL

Case	T_1^*	T_2^*	T_3^*	T^*	Q_{max}^* **(rounded off)**	$\varphi^t(T_1, T_2, T_3)$
Example 1						
1	0.3227	7.2985	21.3194	28.9545	34	194.88
2	0.5302	7.6269	29.5440	37.7011	35	267.05
3	0.5178	1.4836	04.9466	6.948	34	213.85
Example 2						
Case	T_1^*	T_2^*	T_3^*	T^*	Q_{max}^* (rounded off)	$\varphi^t(T_1, T_2, T_3)$
1	0.8341	6.8149	08.7521	16.4011	182	2356.53
2	0.2106	7.0602	09.3465	16.6173	028	2087.43
3	0.4845	1.0090	0.8298	02.3233	084	1012.20

From each Table 2, it is observed that in each case, the total inventory cost and the cycle length are directly proportional to the cost of ordering, i.e. the total inventory cost as well as cycle length with an increase in ordering; both increases.

TABLE 2
VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO C_o

Case	C_o	T_1^*	T_2^*	T_3^*	T^*	Q_{max}^***(rounded off)**	$\varphi^t(T_1, T_2, T_3)$
1	200	0.2871	7.3084	20.1123	27.7078	21	184.29
	500	0.3227	7.2985	21.3194	28.9406	34	194.88
	600	0.3342	7.3081	21.7101	29.3524	24	198.31
2	200	0.5147	7.6329	28.6260	36.7736	33	258.99
	500	0.5302	7.6269	29.5441	37.7012	35	267.05
	600	0.5315	7.6250	29.8452	38.0017	35	269.69
3	200	0.4194	1.3625	03.8423	5.6242	24	166.14
	500	0.5178	1.4836	04.9466	6.948	34	213.85
	600	0.5440	1.5177	05.2708	7.3325	36	227.85

From each Table 3, it is observed that in each case the total inventory cost increases when the cost in RW decreases, the ordering length increases. When the cost increases in RW, both the total inventory cost and the length of the ordering cycle increase.

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TABLE 3
VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO H_r

Case	H_r	T_1^*	T_2^*	T_3^*	T^*	Q_{max}^* (rounded off)	$\varphi^t(T_1, T_2, T_3)$
1	1	0.9340	7.2982	21.3720	29.6042	53	195.40
	2	0.4827	7.2981	21.3742	29.155	31	195.34
	5	0.1921	7.2998	21.1724	28.6643	53	198.10
2	1	0.9771	7.6279	29.4002	38.0052	94	265.79
	2	0.6673	7.6272	29.4997	37.7942	50	266.66
	5	0.3928	7.6266	29.5887	37.6081	22	267.41
3	1	0.9268	1.4693	5.1118	7.5079	86	208.04
	2	0.6458	1.4755	4.9014	7.0227	48	211.88
	5	0.3872	1.4886	4.9937	6.8695	22	215.88

From Table 4, it is observed that in each case the total inventory cost increases when the cost in OW decreases and the cycle length decreases. When the cost rises, the total inventory cost as well as the cycle length increase.

TABLE 4
VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO H_w

Case	H_w	T_1^*	T_2^*	T_3^*	T^*	Q_{max}^* (rounded off)	$\varphi^t(T_1, T_2, T_3)$
1	1	0.2106	7.0602	9.3765	16.6437	9	199.40
	3	0.4858	7.5578	32.4287	40.4723	31	340.78
	4	0.5053	7.4838	38.7984	46.7875	32	348.26
2	1	0.4434	7.4828	19.1624	27.0886	27	175.95
	3	0.5731	7.6808	37.6472	45.9011	39	338.15
	4	0.5957	7.7105	44.5261	52.8323	42	398.52
3	1	0.5178	1.4836	4.9466	6.948	34	213.85
	3	0.3670	0.8445	5.6263	6.8378	20	226.82
	4	0.2443	0.7251	5.7831	6.7525	12	249.97

Fuzzy Model

From Tables 5, 6 and 7, it is observed that in each case, the total inventory cost in the fuzzy model decreases when working with a single fuzzy parameter and with an increase in cycle length.

TABLE 5
VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $C_{\tilde{o}}$

Case	$T_1^{\tilde{}}$	$T_2^{\tilde{}}$	$T_3^{\tilde{}}$	T^*	$fQ_{max}^{\tilde{}}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.3342	7.2955	21.7104	29.3401	24	148.73
2	0.5302	7.6227	29.5441	37.697	35	202.28
3	0.4861	1.0656	5.7151	7.2668	31	185.28

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TABLE 6 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $\mathbf{H}_{r}$						
Case	$\tilde{T_1}$	$\tilde{T_2}$	$\tilde{T_3}$	$\tilde{T}$	$f\tilde{Q}_{max}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.3007	7.2956	21.6888	29.2851	22	148.18
2	0.5302	7.6227	29.5441	37.697	35	202.28
3	0.4861	1.0656	5.7151	7.2668	31	185.28

TABLE 7 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $\mathbf{H}_{w}$						
Case	$\tilde{T_1}$	$\tilde{T_2}$	$\tilde{T_3}$	$\tilde{T}$	$f\tilde{Q}_{max}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.4963	7.4653	36.6172	44.5788	32	246.84
2	0.5898	7.7022	42.3366	50.6286	41	284.45
3	0.2902	0.7596	5.7376	6.7874	14	186.00

From each Table 8, it is observed that in each case, the total inventory cost decreases and the cycle length increases when working with cost and holding cost as fuzzy parameters in RW.

TABLE 8 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $\mathbf{C}_{o}$ $\mathbf{H}_{r}$						
Case	$\tilde{T_1}$	$\tilde{T_2}$	$\tilde{T_3}$	$\tilde{T}$	$\tilde{Q}_{max}$ **(rounded off)**	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.3007	7.2956	21.6828	29.2791	22	148.18
2	0.5036	7.6249	29.8555	37.984	32	202.34
3	0.4570	1.0662	05.7237	7.2469	28	185.56

From Tables 9, 10 and 11 it is observed that in Cases-1 and Cases-2, the total inventory cost increases, while in OWs work with ordering cost and holding cost and as fuzzy parameters There is a combination of three costs and the total inventory in Case-3. Cost reduction but increasing cycle length in each case.

TABLE 9 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO f $\mathbf{C}_{o}$ $\mathbf{H}_{w}$						
Case	$\tilde{T_1}$	$\tilde{T_2}$	$\tilde{T_3}$	$\tilde{T}$	$\tilde{Q}_{max}$ **(rounded off)**	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.4963	7.4653	36.6172	44.5788	32	246.84
2	0.5933	7.7014	42.5612	50.8559	42	300.66
3	0.3303	0.7776	06.0910	7.1989	17	196.75

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TABLE 10 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $H_{r^{\sim}}$ $H_{w^{\sim}}$						
Case	$T_1^{\sim}$	$T_2^{\sim}$	$T_3^{\sim}$	$T^{\sim}$	$Q_{max}^{\sim}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.4400	7.4663	36.6172	44.5235	29	245.09
2	0.5548	7.7021	42.3466	50.6035	37	284.54
3	0.2722	0.7597	5.7401	6.772	13	186.09

TABLE 11 VARIATION IN TOTAL INVENTORY COST WITH RESPECT TO $C_{o^{\sim}}$ $H_{r^{\sim}}$ $H_{w^{\sim}}$						
Case	$T_1^{\sim}$	$T_2^{\sim}$	$T_3^{\sim}$	$T^{\sim}$	$Q_{max}^{\sim}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.4468	7.4653	36.6080	44.5201	29	246.78
2	0.5558	7.7014	42.5713	50.8285	38	286.02
3	0.3105	0.7764	6.0724	7.1593	16	196.86

For each case of crisp model and in Figure 4 the graph shown in Figure 5 is for each case of the fuzzy model show that there is a point where the minimum cost with constraint in the inventory system is that T_1, T_2 and T_3 are all positive.

TABLE 12 FUZZY MODEL						
Example 1						
Case	$T_1^{\sim}$	$T_2^{\sim}$	$T_3^{\sim}$	$T^{\sim}$	**$Q_{max}^{\sim}$ (rounded off)**	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.3418	7.2933	21.9682	29.6033	24	150.43
2	0.5385	7.6237	30.0446	38.2068	36	203.58
3	0.5041	1.0800	05.9240	07.5441	32	192.05
Example 2						
Case	$T_1^{\sim}$	$T_2^{\sim}$	$T_3^{\sim}$	$T^{\sim}$	$Q_{max}^{\sim}$ (rounded off)	$\tilde{\varphi}^t(T_1, T_2, T_3)$
1	0.8418	6.6785	8.8797	16.4000	186	1785.57
2	0.8994	6.5832	8.0728	15.5554	212	1668.72
3	0.4845	1.0090	0.8298	02.3233	084	0759.15

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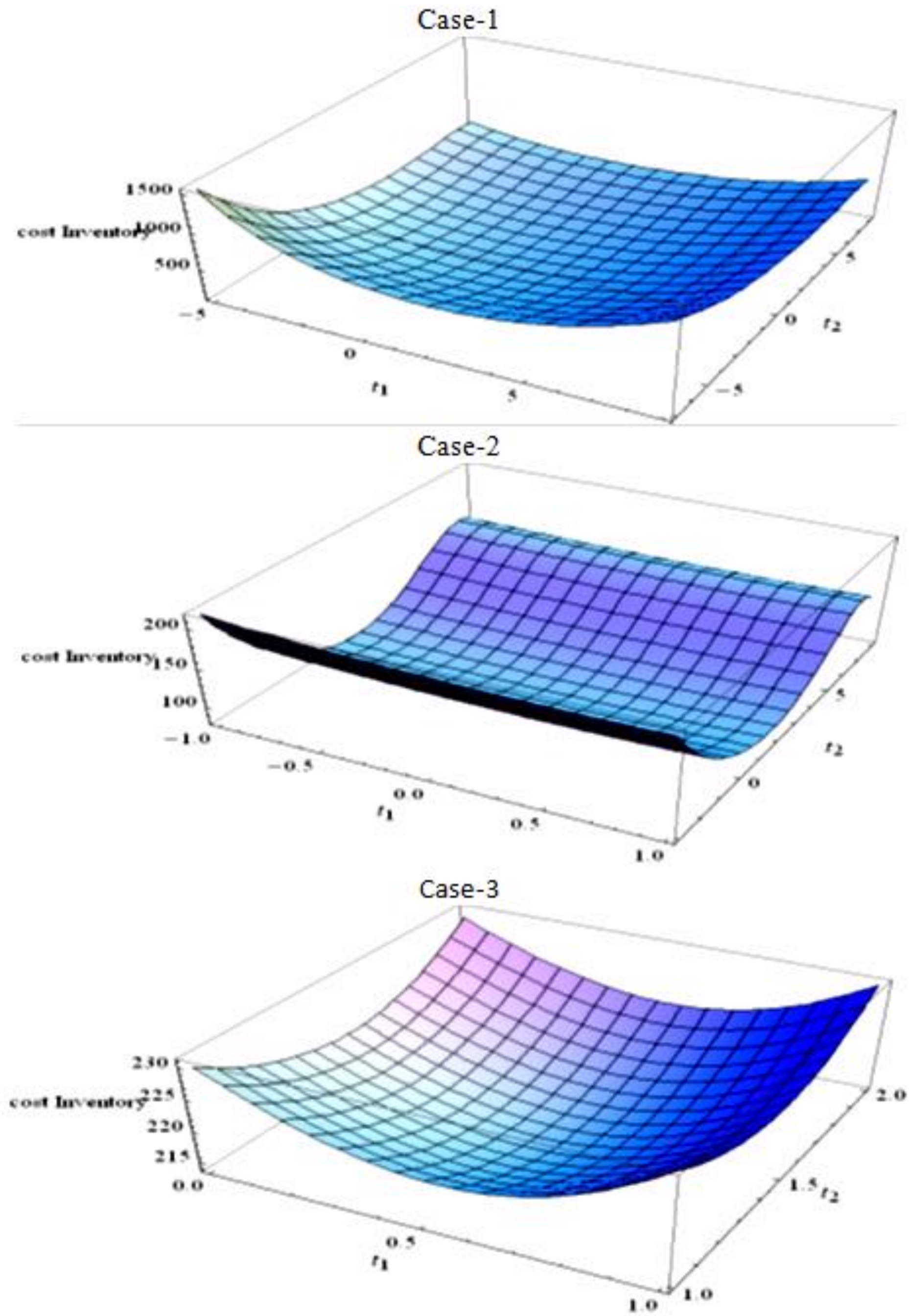

FIGURE 4

GRAPHICAL REPRESENTATION OF CONVEXITY FOR CRISP MODEL

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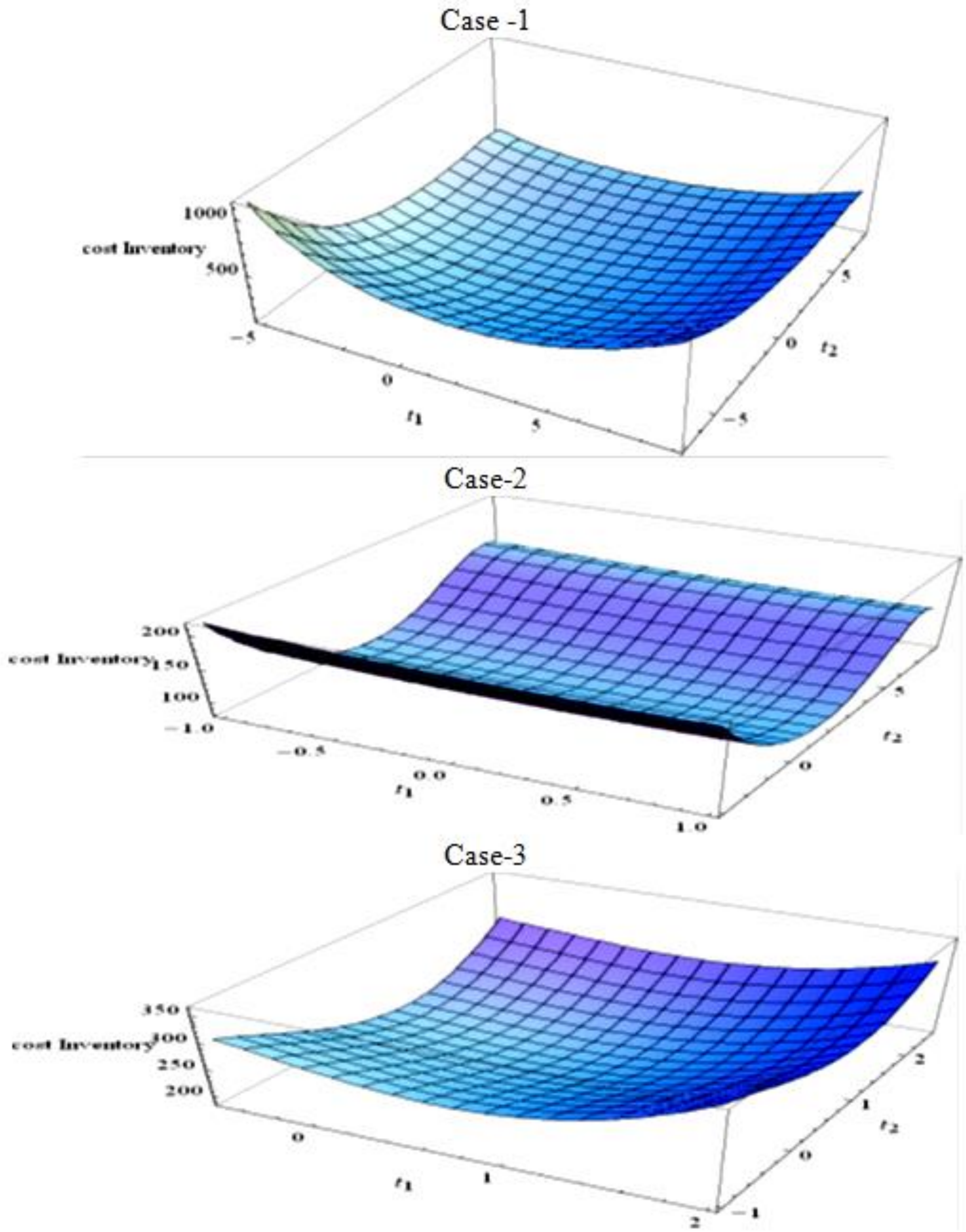

FIGURE 5

GRAPHICAL REPRESENTATION OF CONVEXITY FOR FUZZY MODEL

CONCLUSION

A deterministic inventory mock-up is obtainable to settle on the most favourable inventory cost for the two-warehouse inventory mock-up with ramp type demand, Weibull distribution degradation rate and partial backlog at constant rate. Two models namely a crisp

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model and a fuzzy model have been developed and numerical examples have been presented to illustrate and validate themodels.The results are obtained and compare with the help of appropriate software. Sensitivityanalysis is performed on some chosen parameters. The fuzzy model is solved with the rescue of triangular fuzzy numbers with the help of the signed distance method.In addition, this model can be normalized by taking into account the price-dependent demand, stock-dependent demand, and a constant decline rate with other realistic combinations.

We residential a two-warehouse inventory mock-up for demand rate as a universal ramp-type function of moment and have studied the effect of the two-parameter Weibull distribution degradation rate and fajita with fuzzy triangular numbers.The cost of ordering and holding costs in both ware-houses are treated as fuzzy triangular numbers and the signed distance method is used to defile the total inventory cost. Only one item is included in this study. Shortages are authorized& partially backordered at a constant rate and the results of the crisp and fuzzy models are compared with the help of numerical examples. Sensitivity is also played for both model son limiting parameters.

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Citation Information: Sethi, G., Yadav, A. S., & Singh, C. (2022). Two ware-houses fuzzy inventory model for deteriorating items with ramp type demand and shortages. *Journal of Management Information and Decision Sciences, 25*(2), 1-22.

APPENDIX 1

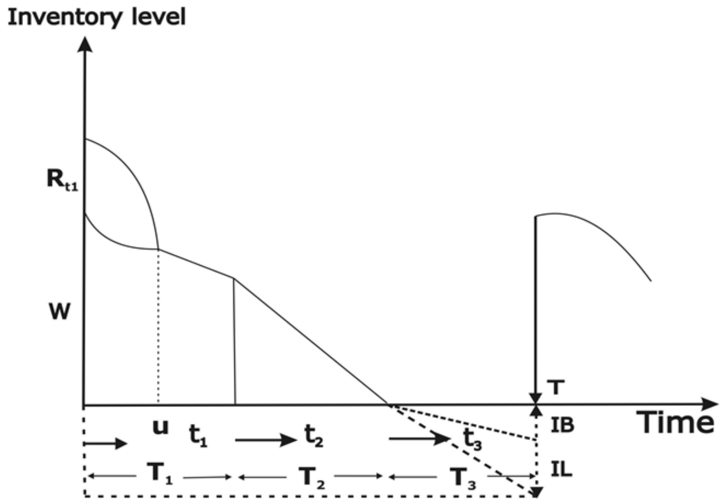

FIGURE A1

INVENTORY TIME GRAPH FOR CASE 1 OF INVENTORY SYSTEM

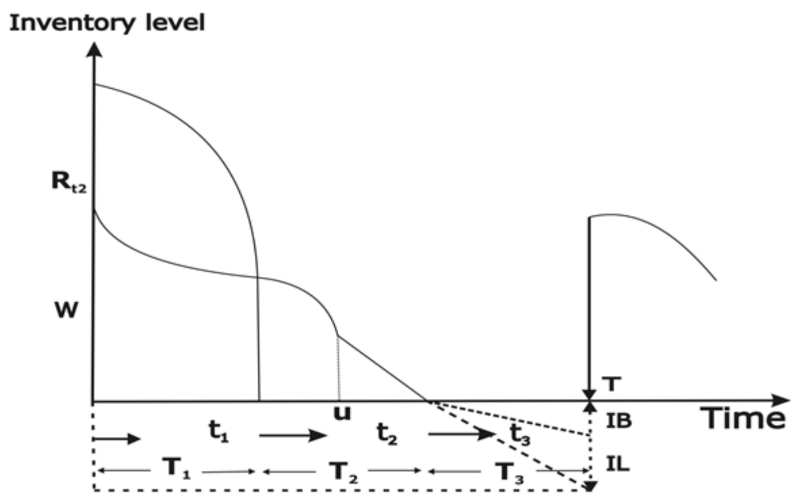

FIGURE A2

INVENTORY TIME GRAPH FOR CASE 2 OF INVENTORY SYSTEM

Citation Information: Sethi, G., Yadav, A. S., & Singh, C. (2022). Two ware-houses fuzzy inventory model for deteriorating items with ramp type demand and shortages. *Journal of Management Information and Decision Sciences, 25*(2), 1-22.

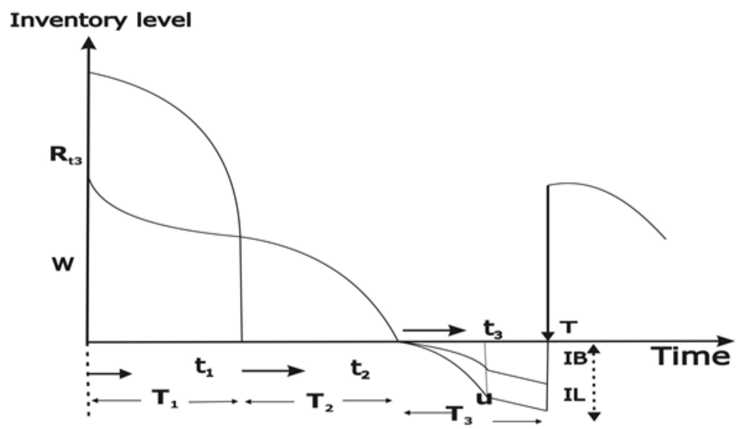

FIGURE A3

INVENTORY TIME GRAPH FOR CASE 3 OF INVENTORY SYSTEM

APPENDIX 2

Table A

Parameter	A	b	u	C_o	S_c	L_c	D_o	α	β	G	h	f_d	H_r	H_w	W
Example-1	30	4.5	0.5	500	0.15	0.20	4.0	0.02	2.0	0.05	2	0.6	3.0	2.0	50
Example-2	100	3.0	1.0	1000	1.0	4.0	2.0	0.03	2.0	0.06	2	0.4	6.0	3.0	100

Citation Information: Sethi, G., Yadav, A. S., & Singh, C. (2022). Two ware-houses fuzzy inventory model for deteriorating items with ramp type demand and shortages. *Journal of Management Information and Decision Sciences, 25*(2), 1-22.