

ASCERTAINING THE EFFICIENT FRONTIER IN THE CLASSROOM

Stephen C. Caples, McNeese State University
Michael E. Hanna, University of Houston—Clear Lake
Grady Perdue, University of Houston—Clear Lake

ABSTRACT

This paper presents a tutorial on how to use the Solver function of Microsoft Excel to teach students in an investments class how to implement much of the theory that is associated with Modern Portfolio Theory. With this tutorial the instructor is shown how to develop the efficient frontier of investments for the class, find the specific asset weightings that produces a target rate of return with the least possible risk, and how to introduce additional constraints into the analysis.

Keywords: *Modern Portfolio Theory, Efficient Frontier, Beta*

INTRODUCTION

One common objective of investments courses is introducing students to the many aspects of Modern Portfolio Theory (MPT). Within MPT students are taught about the efficient frontier of portfolios made up of the set of efficient portfolios. These efficient portfolios are important to the investor because they offer the least amount of risk at each possible rate of return and the best return possible for each level of risk. Any other portfolio will be characterized by higher levels of risk than are necessary to produce any given rate of return.

Most university investments courses deal with MPT—and particularly the efficient frontier—from a theoretical stand point. Unfortunately some students have difficulty following the theoretical discussions and so they cannot fully grasp the powerful practical implications found in MPT. Also there are some students who do understand the theory, but have difficulty making the transition to a practical application in the construction of an actual portfolio. Most textbooks fail to address these instructional problems.

In this study we present a tutorial on how to use a special function in Microsoft Excel to teach students about MPT. Following the approach presented in this tutorial, the instructor can start with a given set of assets and find the efficient (minimum risk) portfolio for a given rate of return. Students can be asked to find efficient portfolios as well because of the ubiquitous nature of Excel. Once the basic portfolio is developed, the results can be modified to present additional constraints on the portfolio to prevent one particular asset from being too heavily weighted.

SETTING THE EXAMPLE

For simplicity in this tutorial, you are only given five assets to build your portfolio. As such your analysis will ascertain the best portfolio available to the investor given the small set of investment possibilities. However, you will recall that in his analysis Markowitz (1952, 1959) assumed a data set made up of *all* of the risky assets in the world. While it is a little unrealistic to assume you could actually assemble that data set for an analysis, it is true that the larger your data set the more options are open to you for building a great portfolio. So the odds are that the larger the data set then the better will be the portfolio ultimately derived from your analysis. If one can perform this analysis for the investor with a five asset data set, then the analysis with a 50 or 500 asset database is the same.

Markowitz originally presented his theory using standard deviation as the risk measure, and he sought to minimize the constrained non-linear equation for the standard deviation of the portfolio to arrive at efficient portfolios. Later research by Sharpe (1964) simplified the modern investor's work as he introduced beta as the measure of systemic risk in a portfolio. Sharpe's discovery of beta allows the investor to minimize the linear equation for the beta of the portfolio, which many more computer software resources can accommodate.

Given below is the initial set of assets with which you will work. Asset B can be thought of as a broad market mutual fund, with the same risk and return as the overall market. The other assets may be other funds or individual investments.

Table 1: Data Set					
	Asset A	Asset B	Asset C	Asset D	Asset E
Beta	1.2	1	0.8	0.6	0.4
Expected Return	11%	10%	8%	7%	6%

The portfolio that you recommend to the investor will be made up of from one to five of these assets. The final portfolio will consist of a certain percentage or weighting of each of these assets, with a weight of zero indicating that the asset is not included in the portfolio. You must solve for the percentages or weights providing the lowest beta and still producing the required rate of return. At the moment these weights are not known. However, this problem can be stated as a linear programming problem and very easily solved in Excel.

The basic statement of the problem is:

Minimize beta

Subject to:

Total return of the portfolio must be at least the specified amount

The sum of the weights must equal 1.

If w_1 = weight of Asset A, w_2 = weight of Asset B, w_3 = weight of Asset C, w_4 = weight of Asset D, w_5 = weight of Asset E, this example problem is stated as

$$\text{Minimize beta} = 1.2w_1 + 1w_2 + 0.8w_3 + 0.6w_4 + 0.4w_5$$

Subject to:

$$0.11w_1 + 0.10w_2 + 0.08w_3 + 0.07w_4 + 0.06w_5 \geq \text{required rate of return}$$

$$w_1 + w_2 + w_3 + w_4 + w_5 = 1$$

All weights must be non-negative

Once the rate of return is chosen, this problem can then be entered into Excel and easily solved.

CONSTRUCTING THE SPREADSHEET

To begin, set up your spreadsheet as shown in Table 2. Cells B1:F1 are where your answers (weights) will appear, cells G4:G6 will hold formulas to calculate the total beta (G4), the total return (G5), and the total of the weights (G6), and the cells B4:F6 and I5:I6 are for input values from the objective and the constraints listed above. In cell G4 enter this *exact* formula: =SUMPRODUCT(\$B\$1:\$F\$1,B4:F4) to calculate the total beta. This multiplies the weights in row 1 by the betas in row 4 and adds them together. Copy the formula in G4 into cells G5 (total return) and G6 (total of the weights). It is helpful to put a 1 in cells B1:F1 at this point to check your formulae, although this is not shown in Table 2.

	A	B	C	D	E	F	G	H	I
1	Solution (weights)								
2									
3	Asset	A	B	C	D	E	Total		Limits
4	Beta	1.2	1.0	0.8	0.6	0.4			
5	Return	0.11	0.10	0.08	0.07	0.06		>=	.06
6	Weights sum to 1	1	1	1	1	1		=	1.000

Examine what you have so far by looking at each row separately.

1. Row 1 is where the solution (i.e., the weightings of each asset) will appear.
2. Row 2 is simply blank space to set off the solution you will find.
3. Row 3 names the five assets in the data set placed within columns. Information about each asset will be placed in its column in the subsequent rows.
4. Row 4 reports the beta of each asset.

5. Row 5 shows the expected return of each asset in Columns B through F. In Row 5, Column H you see a mathematical sign (for reference only) and in Column I you see the value .06 indicating you want a return equal to at least six percent from these assets. (The reason the ">" is included is that if a return of six percent is not feasible, you want the closest possible return greater than six percent.)
6. Row 6 indicates that weights in the final portfolio must sum to "1."

ADDING/ACTIVATING SOLVER

To find the solution, you will use Solver which is an add-in that comes with Excel. At the top of the Excel worksheet, click on the "Data" tab. Look for "Solver" on the Ribbon at the top of the worksheet. If you do not see it, you must activate it. If you do see Solver on the Data tab, then you can skip the next set of steps where adding/activating Solver is described.

To activate Solver:

1. In Excel 2010, go to the upper left corner of your Excel page and click the File tab. In Excel 2007, click on the round (and multi-color) Microsoft Office button
2. Click on "Excel Options" and then click on "Add-Ins."
3. At the bottom of the window, locate the "Manage" dialog box and select "Excel Add-Ins" from the list. Click "Go."
4. Check the box for the Solver Add-in and click "OK." You should see Solver on the Data Tab in the Excel Ribbon. (Be patient if the installation takes a few minutes.)

THE EFFICIENT FRONTIER

Now that you have Solver on the Data tab, you can use it to find the efficient frontier. In this section of the paper we will find several points on an efficient frontier generated from the data set given above. In this analysis we shall use beta as the risk measure of the assets in the data set and the portfolio.

To find the efficient frontier, solve for the efficient portfolios that would have required returns of six, seven, eight, nine, ten and 11 percent, respectively. These must be solved for individually. First we shall solve for the portfolio that produces the six percent return, and then move on to the other required portfolio returns. Do the following:

1. Click on Solver. A window labeled "Solver Parameters" now pops up.
2. For Excel 2010 only: check the box for "Make Unconstrained Variables Non-Negative," and choose Simplex LP in the window for "Select a Solving Method."

3. For Excel 2007 only: click “Options.” When the window for “Solver Options” opens, check the boxes marked “Assume Linear Model” and “Assume Non-Negative.” Now click on “OK” and you return to the previous dialog box.
4. The cell that contains the value (total beta) to be minimized is called the Objective in Excel 2010 and the Target Cell in Excel 2007. Type G4 into this Objective or Target Cell box (or click this box to highlight it and then click on cell G4).
5. Click the button that says “Min” to specify that the Objective/Target Cell is to be minimized
6. Click on the box “By Changing Variable Cells” (the word “Variable is omitted in Excel 2007) and select cells B1 through F1. This is where the final weightings of the assets will appear. The Objective/Target Cell is to be minimized by changing the values in these cells (which contain the weights).
7. Click on “Add” and then designate cell G5 (total return), then select “>=,” then select cell I5. This tells the program the portfolio return (G5) must be equal to or greater than 6.00 percent (I5).
8. Click “Add” again and select cell G6, click on “=,” then select cell I6. That constraint tells the program that the sum of the weights (G6) must equal one (I6).
9. Click on “Solve” and optimum asset weights appear in cells B1 through F1.

The portfolio that was just found is a single point on the efficient frontier and is the portfolio that produces the six percent rate of return. This portfolio is described on the first row of the following table. The portfolio is entirely made up of Asset E, which is the asset with a six percent rate of return. Other points on the frontier may be found using the process just described, but substituting in other possible returns into cell I5 at the start of the search for each new portfolio. The results will be:

Required return	Beta	Asset A	Asset B	Asset C	Asset D	Asset E
.06	0.40	0%	0%	0%	0%	100%
.07	0.55	0%	25%	0%	0%	75%
.08	0.70	0%	50%	0%	0%	50%
.09	0.85	0%	75%	0%	0%	25%
.10	1.00	0%	100%	0%	0%	0%
.11	1.20	100%	0%	0%	0%	0%

What has been developed here is the efficient frontier. Each portfolio is efficient in that no other portfolio can be constructed from this data set that produces the same returns and does

so with lower beta measures of risk. For example the minimum risk portfolio that produces an eight percent return does so with a beta of 0.70 and equal weightings of Assets B and E in the portfolio. The other three assets are omitted from the efficient portfolio. It is not possible to construct a portfolio from this set of assets that produces an eight percent return and does so with less risk (i.e., a smaller beta) than this portfolio.

THE TARGET PORTFOLIO

Now that the efficient frontier has been derived, let's help the investor find his target portfolio. Assume that the investor is constrained to this set of assets, and the investor wants the minimum risk portfolio that would produce a portfolio return of 8.74 percent. First change the value in cell I5 to read "0.0874" so that you can solve for the efficient portfolio that will produce that return. Then activate Solver and solve for the solution. Below is the solution you should find in cells B1:F1.

Asset A	Asset B	Asset C	Asset D	Asset E
0%	68.5%	0%	0%	31.5%

The investor's optimum (minimum risk) portfolio is made up of only assets B and E, weighted with 68.5 percent of funds in Asset B and the remaining 31.5 percent of funds placed into Asset E. This target portfolio solution provides you a final portfolio beta of **0.8110**. You cannot put together another portfolio with these five assets that will give you a return of 8.74 percent with a smaller beta (i.e., less risk). Have your students try and see if they can prove the solution wrong.

THE CONSTRAINED TARGET PORTFOLIO

Now take this analysis one step further. Assume that you are not happy advising the investor to put more than 2/3 of the money into one asset. What can you do? The answer is that you can limit the maximum weighting of any asset in the final portfolio. Suppose you wish to limit the weight of any single asset to no more than 35 percent of the total portfolio. You can limit the weight of each asset by adding additional constraints. For example, $w_1 \leq 0.35$ would limit the weight for Asset A., and constraints on the weights for the other assets would be formulated in a similar fashion. The Excel worksheet would be modified as follows:

1. Put "1" in cells B7, C8, D9, E10, and F11, as shown in Table 5 for each of the five new rows.
2. Put "0.35" in each of the cells I7 through I11.
3. Copy cell G4 into cells G5 through G11.

4. Click on the Solver button again. Note that the original problem is still there.
5. When the dialog box opens, add a new constraint by clicking on “Add.”
6. Designate in the proper spaces the cell reference of “G7:G11.” You can just click G7 and drag the cursor to G11.
7. Select the “<=” sign.
8. To the right of the “<=” sign, enter “I7:I11”.
9. Click OK.
10. Click Solve.

	A	B	C	D	E	F	G	H	I
1	Solution (weights)								
2									
3	Asset	A	B	C	D	E	Total		Limits
4	Beta	1.2	1.0	0.8	0.6	0.4			
5	Return	0.11	0.10	0.08	0.07	0.06		>=	0.0874
6	Weights sum to 1	1	1	1	1	1		=	1.000
7	weight for A	1	0	0	0	0		<=	0.35
8	weight for B	0	1	0	0	0		<=	0.35
9	weight for C	0	0	1	0	0		<=	0.35
10	weight for D	0	0	0	1	0		<=	0.35
11	weight for E	0	0	0	0	1		<=	0.35

The revised solution uses four of the five assets, and still produces an expected return of 8.74 percent. The beta of this new optimum portfolio is **0.8260**. The beta is slightly larger because your limitation of the weight of each asset and their constraints in Excel caused the previous solution to be unusable. Notice that Asset B comprises only 35% of the portfolio instead of 68.5% as before in Table 4.

Asset A	Asset B	Asset C	Asset D	Asset E
26%	35%	0%	4%	35%

Given the *very* marginal difference in the betas of these two portfolios, you can certainly argue that the investor is probably better off with this second (more diversified) portfolio.

REFERENCES

- Markowitz, H. 1952. Portfolio Selection. *Journal of Finance*. VII (1): 77-91.
- Markowitz, H. 1959. *Portfolio Selection—Efficient Diversification of Investments*. New Haven: Yale University Press.

Sharpe, W. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. Journal of Finance. XIX (3): 425-442.

AUTHOR PROFILES:

Stephen Caples earned his Ph.D. at the University of Texas—Arlington in 1985. Currently he is a professor of finance at McNeese State University.

Michael Hanna earned his Ph.D. at the Texas Tech University in 1981. Currently he is a professor of decision sciences at the University of Houston—Clear Lake.

Grady Perdue earned his Ph.D. at the University of Alabama in 1985. Currently he is a professor of finance at the University of Houston—Clear Lake.