DEVELOPING THE PHYSICAL DISTRIBUTION DIGITAL TWIN MODEL WITHIN THE TRADE NETWORK

Sergey Yevgenievich Barykin, Peter the Great St. Petersburg Polytechnic University
Irina Vasilievna Kapustina, Peter the Great St. Petersburg Polytechnic University
Sergey Mikhailovich Sergeev, Peter the Great St. Petersburg Polytechnic University
Olga Vladimirovna Kalinina, Peter the Great St. Petersburg Polytechnic University
Viktoriia Valerievna Vilken, Peter the Great St. Petersburg Polytechnic University
Elena de la Poza, Universitat Politècnica de València
Yuri Yevgenievich Putikhin, Financial University under the Government of the Russian Federation
Lydia Vitalievna Volkova, Financial University under the Government of the Russian Federation

ABSTRACT

The concept of digitalization is based on the dominance of digital ecosystems and on the widespread introduction of artificial intelligence systems including the physical distribution within the trade networks. The implementation of internet of things and artificial intelligence as well as machine learning allows implementing the digital twins. From an economist's point of view, the digital twin does not make sense without a mathematical model. The study aims at the developing a physical distribution digital twin model being focused on its use in managing trade network activities in conjunction with information cyberspace. The research subject of this paper includes the methodology of the physical distribution formation within the trade network. So, the authors suggest the definition of the investigated physical distribution digital twin (economic) model as a software implementation of the algorithm for processing a set of physical distribution data arriving online and at each moment of time reflecting the activity of the considering trade network. During the research, methods of stochastic modelling and dynamic programming as well as mathematical methods of optimization were used. The results obtained can be incorporated into the algorithmic basis of digital platforms of the trading network ecosystem. The article examines the features of the physical distribution digital twin models from the economic point of view. Taking into account the proposed definition of the digital twin, the future scientific research can cover the development of scientific discussion on questions of optimization methods for the development of the physical distribution within the trade network.

Keywords: Digital Twin, Smart Supply Chains, Physical Distribution, Trade Network Management, Digital Logistics, Economic Model

INTRODUCTION

The globalization of economic activity today determines not only the structure of business in the area of trade. The principles of network organization dominate general services, production, and areas relating to the organization of logistics, courier services, healthcare, recreation, and areas, which provide security and safety in the whole world. Since the activities of network business are headed from a single management centre (head office), top managers face a number of problems:

- A uneven consumer landscape surrounding each division of the network
- Varying returns on investment for businesses of the trade network
- A frequent change of goods offered to consumers, making up an assortment matrix
- The factor of market uncertainty caused by consumer preferences and by the activities of competitors in this business sector.

The research subject of this paper includes the methodology of a trade network formation, so the second definition relates to the topic of the study. The purpose of the research consists of improvement of the developing a mathematical model being focused on its use in managing trade network activities in conjunction with information cyberspace. The research task could be described as follows. Due to the fact that modern business is highly dynamic, there is a need to have stable feedback from each of the enterprises of a commercial network and, most importantly, the chance to make optimal decisions in the least amount of time. It is the possibility of digital interaction itself which allows real-time information to be received online about the movement of goods, volume of demand, and financial indicators using a single computer database. This data is accessible from POS (point of sale) terminals working offline with physically present consumers, and from online interactions statistics. Likewise, with the introduction of the concept of digital transport corridors and the corresponding machine-readable marking, information about the whereabouts of goods and cargo is also presented in real time. This makes it possible to transition to predictive analytics and predictive planning of JIT (just-in-time) management as part of the management model for uninterrupted supply chains.

All of this data is included in the array of information offering feedback. The digital transformation has led to a specific approach of the simulation-based planning and optimization concepts being considered as the digital twin (Kritzinger et al., 2018).

Digital twins are made possible due to the implementation of such technologies as Internet of Things (Atzori et al., 2010; Shen & Liu, 2011) and artificial intelligence as well as machine learning. Digital twin updates itself using data from sensors or external entities. Some researchers consider digital twin as a digital replica of business entities (Fuller et al., 2020; Tekinerdogan & Verdouw, 2020). Digital twins could be understood as up-to-date digital representations of the physical and functional properties of a system (Sacks et al., 2020).

The concept of digitalization is based on the dominance of digital ecosystems and on the widespread introduction of artificial intelligence systems, which have at their base computer programs which implement such algorithms. From an economist's point of view, the digital twin does not make sense without a mathematical model.

Within the framework of this task, the economic meaning of the physical distribution digital twin model within the trade network could be as follows: it is a software implementation of the algorithm for processing a set of physical distribution data arriving online and at each moment of time reflecting the activity of the considering trade network.

The result of the algorithm could be considered as a predictive modelling of the dynamics of the development of the physical distribution within the trading process. After solving the optimization problem, recommendations are formed to support the decisions of the trade network managers. Other factors also exist which have to do already with specific regions of activity as well as the rate at which digital technologies have penetrated various sectors of economic activity (Barykin, Kapustina, et al., 2020; Kapustina et al., 2020).

MATERIALS AND METHODS

Theoretical Fundamentals and Methodology

It is advised to look at the problem of digital twin modelling from the perspective of the possibilities of simulation and optimization models (Zhang et al., 2019).

Optimization, artificial intelligence, or other advanced analytics can be part of an effective supply chain digital twin. A digital twin as part of a wide range of control tower systems provides end-to-end supply chain management (AnyLogistix, 2020).

The value of the digital twin of the trade network lies in the continuous improvement of short-term and med-term solutions.

• Mid-term decisions are mainly related to how the sales network should operate, including design and planning of activities. The digital twin allows improving trade network structure and core processes, as well as resources and optimization logic.

• Short-term solutions are mainly concerned with identifying potential problems and analysing solutions, such as planning transportation or quantifying the bull whip effect from an external disturbance.

As far as the possibility of making optimal decisions when managing a trade network, this data becomes the arguments for the mathematical model. The main objective is to come up with algorithms for management decision making with regard to trade networks. Developing a set of formalisms at the basis of an adequate mathematical description solves a number of very important problems in the realm of managing trade networks. This makes it possible to transition to the algorithmization of specifically these kinds of digital twins:

- Possibility of end-to-end support of any commercial transactions
- Strategic planning using innovative methods based on leading economic indicators
- Use of well-formalized methods of the mathematical theory of stochastic management to search for solutions to economic problems during market uncertainty
- Maintaining the maximum profit level when several generations of products are on the market and their interchange ability with innovative offers
- Real-time interaction with dynamically changing consumer demands with consideration of the socio-demographic landscape
- Modelling stress tests which allow the high indicators for the sustainability of business solutions and their effectiveness to be maintained.

All nodes of a trade network are joined into a single financial and economic structure. For each of them, a contact surface is determined, acting as the interface of interaction providing the synchronization of physical and information exchange within trade activities. When creating the mathematical model, a selection of input and output parameters as the data vector for the interface of this physical node is clearly selected. At the same time, a number of possible interlink ages are determined in the form of a graph which determines the orientation and nature of interaction as well as the required extent during these processes in the form of distribution functions.

Mathematical modelling (Krasnov, Sergeev, Titov, et al., 2019) above all reflects the system of information flow in a trade network. By searching for the optimal algorithms to process these flows, market competitiveness is achieved (Borisoglebskaya et al., 2019). The conditions for conducting each kind of business are dictated by the current laws which then formally determines the restrictions when creating the model. Likewise, the quality characteristics of the equipment used in the nodes and the capacity of the interaction channels as well as time factors can serve as restrictors.

The goal of the research is to develop algorithms for management decision of the physical distribution making on the scale of trade networks. The main requirements are optimization according to accepted criteria, as well as the easing of their implementation on digital platforms as part of complexes of management programs (Balaban, 2019) and expert systems for supporting management decisions at various levels of management.

In our study the algorithmization methodology for the physical distribution digital twin within the trade network could be studied taking into account different points of view regarding economic and mathematical issues.

LITERATURE REVIEW

A digital twin is a detailed simulation model of a real-world entity or system which uses real-time data to predict its dynamics and enable understanding, learning and reasoning (AnyLogistix, 2020). In other studies, the digital twin is seen as the digital footprint of a product throughout its life cycle. Digital supply chains-represent the transition from linear, sequential supply chain operations to an interconnected, open ecosystem to build a digital twin, the following components should be used: a standardized information model, high-performance data processing, and industrial communications for collaboration (Liu, et al., 2020). Digital twins use data and computational methods; therefore, it is advisable to base the development of digital twins on surrogate models that have been successfully developed on the basis of the last four decades (Chakraborty et al., 2021).

The development of a special standard for the intelligent production of digital twins can systematize and streamline work in this direction. The digital twin problem was considered by various researches (Artemov et al., 2019; Lu et al., 2019). Digital twins used in the production process can monitor production in real time and especially the efficiency of operators (Ruppert & Abonyi, 2020). This technology makes full use of the digital space parallel to the physical space. It expands the ability of human operators to understand, explore, and control the production elements and processes (Zhuang et al., 2021). Digital twins, designed for logistics chains, take into account their own characteristics. This tool includes object location models, linear mixed integer optimization models, and dynamic modelling techniques for performing multi-scenario "what if" analysis (Marmolejo-Saucedo, 2020). The digital twins of the supply chain are different from the digital twins of other physical objects and processes. This is due to the peculiarities of the logistics configuration of supply chains, as well as the specifics of the relationship between the virtual model and the supply chain in the real world (Barykin et al., 2020). The supply chain management framework, based on the Internet of things technologies, allows you to track the activities of retail chains at any scale (Abdel-Basset et al., 2018). Research shows that digital twins are used in all industries. The most common use of digital twins is noted in manufacturing, energy, construction and aviation (Errandonea et al., 2020). In the modern world, there is an increasing flow of digital technologies such as big data, artificial intelligence, the Internet of Things (IoT), next generation networks, robotics, and social networks, which are used to transform business operations by simplifying such tasks as joint work, involvement and use of information and knowledge (Sutherland, 2018). The exponential growth of data digitization is the basis of the fourth industrial revolution. The key success factor is the ability to react sensitively and quickly to the development of digital technologies and their application in various spheres of life, carrying out the necessary internal and external changes.

The future of digital transformation involves not only serious technological, but organizational and even cultural and mental changes (Tolstykh et al., 2019). Technologies used in the Industry 4.0 have a tendency to reduce exposure to such risks as the risks of changeable customer requirements, satisfaction of the increasingly individualized client needs, as well as operational risks such as mistakes caused by manual processes or equipment malfunctions (Zimmermann et al., 2019).

In accordance with the Gemini Principles creating a national digital twin as an ecosystem of connected digital twins allows to increase the value by using data for the public good (Bolton et al., 2018). The special feature of the digital twin could be defined as its connection to the physical twin, what distinguishes a digital twin from any other digital model (Brenner & Hummel, 2017). Any changes made to the production parameters of the physical system are modelled into new variables and predict the necessary optimizations. Actions can be visualized and assessed without any financial risks or human injuries that cannot be avoided in real production (Malik & Brem, 2021).

The study (Wright & Davidson, 2020) analyses the advantages and disadvantages of using digital doubles. Based on data from the physical asset or system, a digital twin unlocks value principally by supporting improved decision making, which creates the opportunity for positive feedback into the physical twin. According to Gemini principles (Bolton et al., 2018) the digital twin could be considered as a model of an asset or a model of a system:

• Digital twin 1: A dynamic model of an asset, with input of current performance data from the physical twin via live data flows from sensors; feedback into the physical twin via real-time control (regarding this issue the term “smart asset” is used referring to conducting asset management activities with high performance (Xie et al., 2020)).

• Digital twin 2: A static strategic planning model of a system, with input of long-term condition data from the physical twin via corporate systems; feedback into the physical twin via the capital investment process.

METHODOLOGY

In order to formalize the problem of creating algorithms for decision making in the area of trade network management, a number of basic concepts are defined. A hybrid scheme that combines a simulation model and a mathematical programming model is considered in a work (Aranguren et al., 2018) for designing logistic networks for co-firing biomass, specifically switchgrass, in conventional coal-fired power plants. The considering network consists of three sets of nodes: the first set of nodes represents the parcels, the second set corresponds to the depots (a facility to wrap, storage, and consolidate biomass bales), as well as the third is the power plant. Some researchers proposed a hub-and-spoke model for optimizing biomass supply chains for co-firing (Roni et al., 2016). Campbell, J.F. and O’Kelly, M.E. described various fields of Hub-and-spoke networks applications such as terrestrial transportation, airlines, network design, telecommunications (Campbell and O’Kelly, 2012). The work (Strubelt et al., 2018) deals with the optimization of logistics processes at an underground waste storage site and the improvement of a logistical system’s performance using machine scheduling approaches with the support of a plant simulation model. The international trade network cooperation from the multi-distance perspective under the backdrop of the Belt and Road initiative is examined in (Fu et al., 2018).

A trade network is understood as a physical network in which the main component of nodes are any commercial entities operating under the name of a single brand while respecting the principle of unified management, determining at the top level strategy and market policy. These nodes can be distribution centres, trading floors, manufacturing businesses, warehouses, transport terminals, as well as the technical infrastructure for machine-to-machine exchange of information, data centres, and the computing power of management servers and hosting.

Since there are often various standards for sources and presentation of data in the practical interaction of branched commercial structures, depending on country and location, it is necessary that they align.

The second defining concept is the network information add-on. Its function is directly connected to the activities of a physical network. An information network operates the tracking of all operations and records the financial component of each transaction.
The third concept regards the sustainability (Gruchmann et al., 2018) implement an interesting approach to logistics network being constructed to apply for certain sustainable logistics strategies. This work synthesizes the empirical results with regard to the 1PL to 5PL logistics businesses classification scheme and related retention strategies. From the point of view of Logistics Business Transformation for Sustainability, the researchers describe the logistics network consisting of specific facilities, in particular plants, warehouses, distribution centres (DCs), and customers.

The considered trade networks have some similarities with logistics networks the requirements of green economy should be also taken into account. It could be agreed with the thesis that supply chain collaboration and integration, as well as the integration of sharing economy solutions and new digital technologies, have been identified through the qualitative content analysis approach (Gruchmann et al., 2018). So, in our study methodology for the algorithmization of a trade network digital twin could be implemented together with the content analysis being implemented in the abovementioned research.

Progressive shifts in the formation and development of trade networks has led to the introduction of a new approach to managing the system for the movement of goods based on the transformation of the sales model, which is a chain of consecutive actions aimed at optimizing (Podvalny et al., 2017) the process of management decision making. The competitive market environment creates a demand for innovative expert systems to support the management (Pilipenko et al., 2019) of network commerce. A simplified version of this diagram looks as follows (Figure 1).

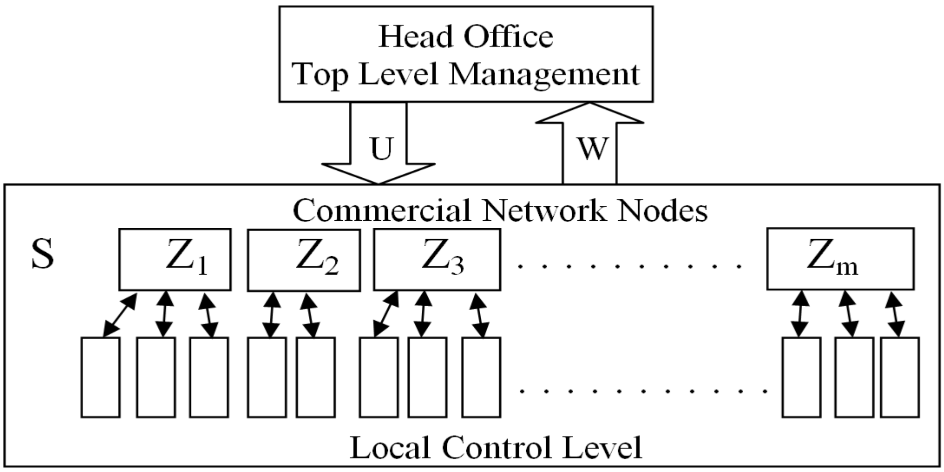

FIGURE 1
THE MANAGEMENT LEVELS OF THE PHYSICAL DISTRIBUTION WITHIN THE TRADE NETWORK.

RESULTS

Developing an Algorithm

The development of scientific knowledge in the area of management makes it possible to introduce the concept of multi-level management, which is based on leading planning indicators as a practice-oriented approach when organizing the budgeting and movement in supply chains. The trend mentioned is characteristic of the whole global retail market.

The most characteristic defining points at the current stage which help evaluate how the presented problem is solved are determined. This is a clearly defined set of indicators for the work of a trade network realized in a unified interface. Software implementation of the algorithm should also allow multi-platform computing since the various management levels use the most diverse equipment from powerful servers to tablets and smartphones. M2M interaction likewise takes place using a broad class of technical solutions such as QR (quick response) code scanners, RFID (radio frequency identification) tags, GNSS (Global Navigation Satellite System), GPS (Global Positioning System) tracking, and many others according to standardized protocols. One of the most important of these is the scalability requirement of the algorithm. This is dictated not only by the cost considerations when expanding a business but also by the constant evolution of business processes, i.e., reengineering. Seeing as the migration of legacy systems is unavoidable, it is necessary to preserve the overall architecture, provide an adaptation to increasing the number of network nodes, and make its typology more complex. By using this approach, we are fully able to reveal the possibilities of interoperability of the contact surfaces, thus minimizing the need to adapt the interface of the network nodes.

Mathematical Modelling

For the purpose of developing a digital twin of the processes taking place in the trading network, we use the representation of its activities in the form of a multi-level process (Borisoglebskaya et al., 2019). This research considers two of these levels the most important when formalizing the operation of a trade network. The first of these, or the one higher in the management hierarchy, the management of the second level receives funds for the planning period and can distribute them with maximum profits.

At each of the two levels listed, a mathematical model is built according to the following limiting conditions:

- A planning horizon is determined ahead, at the end of which a final evaluation of the economic performance indicators is formed;
- The total length of the planning horizon is divided into smaller planning periods or stages;
- According to activity results, an interim evaluation of the economic performance indicators is formed for each of the interim periods;
- Based on the interim evaluations of the economic performance indicators, the management makes management decisions for the following interim planning period;
- A set of key indicators, formalized as a vector which describes the current state of the trade network, is determined;
- The commercial network is clearly presented as a vector of the current state and as a vector which reflects the set of management decisions made by the management at the start of every interim period.

The job of management is to determine a set of management decisions before the start of each interim stage so that the final evaluation of the economic performance indicators is maximized. This means that the quality criteria for management decision making takes into account performance indicators for all of the interim periods. This concept is in line with the current principle of transitioning planning to a system of leading economic indicators.

The mathematical model should be built with consideration of how discrete the data on the activities of the economic subjects and, accordingly, the function arguments is. In order to solve this class of problems, it is advised to use corresponding methods, which are based on the discrete maximum principle developed by Lev Pontryagin. The possibility of using Bellman equations is also considered.

Digital Twin Mathematical Formalization

Algorithmization within the digital twin is necessary for the application of forecasting methods and finding optimal solutions. The formalization of the problem allows us to use the economic nature of the parameters of a real trading network as arguments of a mathematical model and, with the help of optimization theory, to obtain logically proved recommendations for management.

The concept of the system denoted by the symbol is introduced for trade networks. The state of the system is clearly determined by a set of indicators. Depending on the time, the quantitative values of the indicators can change, fully corresponding to the current state of the system. Many such acceptable states are described by the phase space. The top management of a trade network, otherwise system, has the ability to influence the work of the network by making several management decisions. At the same time, the management quality is evaluated by the criterion (Zhabko, Shindyapin, et al., 2019). Since the value is determined by the decisions made by the managers, there is a dependency:

$W = W(U)$

The task of the trade network management is formulated as the search for a set of management decisions by which the quality criteria [17] is at a maximum:

Moreover, there is a limit on the area of initial states S_0 corresponding to the moment in time $t = 0$ and the area of final states S_{FIN}, respectively, as:

$S_0 \in \tilde{S}_0$; $S_{FIN} \in \tilde{S}_{FIN}$, where it is indicated: where $\tilde{S}_0$ is the area of the phase space (Provotorov *et al.*, 2017) at $t = 0$, and $\tilde{S}_{FIN}$ is accordingly the set of permissible states at $t = t_{FIN}$. Likewise, according to the assumptions made the planning horizon from t to t_{FIN} is divided into n interim periods. At the end of each of them, the value W_i is calculated for $i = 1 \ldots n$. It is accepted then that the final value of W for the whole planning horizon is calculated by the additive criterion formula:

$$W = \sum_{i=1}^{n} \theta_i W_i$$

Generally, the values are $\theta_i = 1 \ \forall i = 1,.., n$; however, since as a rule the activities of commercial subjects are evaluated using financial indicators, the inflation component and possibility of reinvestment, the criterion takes into account discounting factors θ_i.

Next, U_i is introduced, where $i = 1 \ldots n$, sets of management decisions for each of the n interim periods. It is determined that the change of state S_{i-1} into the state S_i is determined by a set of managementdecisions U_i and by the preceding state S_{i-1}, in other words, it gives the relation:

$$S_{i-1\,i} = S_{i-1\,i}(S_{i-1}, U_i) \qquad (4)$$

This restriction is not critical since when there is no aftereffects, it is enough to increase the dimension of the phase space S (Artemov *et al.*, 2019) by introducing into it the state parameters for the previous period. If the given terms are used for the formulation, then (3) and (4) can be used to determine the expression (1) as such:

$$W = \sum_{i=1}^{n} W_i(S_{i-1}, U_i)$$

In this way, the goal of the set task is to search for a set of management decisions U_i^*, and in the form of a vector: $U_i^* = \{U_1^*, U_2^*, \ldots, U_n^*\}$, the actions of which, within a trade network or in the terms of the mathematical model (Zhabko, Nurtazina, *et al.*, 2019), transfer the examined system S from the initial state (start of planning period) S_0 into the final S_{FIN} state through n interim periods. The use of such a concept of mathematical formalization of the planning process will make it possible to create an algorithm based on stable feedback with a digital data stream reflecting the economic indicators of current activities.

Making Decisions Method

The technique for using mathematical modeling and the practical calculation method is presented in the following example. Suppose a trade network is made up of m nodes. The subjects constituting the trade network can be retail and wholesale trade and production units, distribution centers, warehouse complexes. Each of them is made up of an internal infrastructure with a standardized interface that provides a contact surface. The top management administers funds to support and develop the work of the trade network. The amount of the funds is equal to z. This amount needs to be allocated to each of the m divisions in the amount of Z_k, where $k = 1, 2, ..., m$.

Z_{1k} Funds are allocated, where 1 is the number of the period, k is the number of the trade network node. Then, it is possible to write down the distribution as a vector: $\overline{Z_1}$, whose dimension is determined by the number of businesses m: $\overline{Z_1} = (Z_{11} .....Z_{1m})$. Since the profit is evaluated after the end of the first period, for the invested amount z, the profit value is written as $f_k(z)$ for each of the $k = 1...m$. It is taken into account that the invested funds are partially spent on current costs, which can be written as: $\phi_k(z) \le z$ for each of the $k = 1...m$. For the second interim period, the allocation of funds is written as $\overline{Z_2} = (Z_{21} .....Z_{2m})$. This operation continues until the last n -th interim period. The task of management is formulated as: the necessity to find a way to optimally manage existing funds.

Choosing Optimality Criterion for Managerial Decision Making

Several popular management options are examined along with their corresponding criteria.

Option 1: The profits obtained from activities are fully invested in further development. The optimization criteria are the maximum sum of the current remaining funds and the profits achieved for the period.

Option 2: The profits obtained from activities are fully invested in further development. The optimization criteria are the maximum of only the profits achieved for the period.

Option 3: The profits obtained from activities are not fully invested in further development but only a specified part of it. The optimization criteria are the maximum of only the profits achieved for the period.

The three options presented have been chosen as the most common. In practice, there can be other criteria lending itself to formalization. The result of the calculation using the mathematical model (Tuegel *et al.*, 2011) (Weyer *et al.*, 2016) should be a set of management decisions $\overline{Z_i}$ for $i = 1...n$, which maximizes the corresponding criteria W. This is tantamount to the fact that in practice trade network management has sound recommendations for redistributing existing funds among the businesses within the network. The result of this activity ensures the maximum for the chosen criteria W.

Digital Twin Lower Level

Mathematical formalisms for the level of a trade network node are similar to those common for the system S . Each of the businesses has an internal structure as well. For example, a retail complex has its own divisions like production, warehouse infrastructure, engineering and energy structure, transport and loading terminals, safety, security, information computing, etc. Likewise, there are affiliated activities within the trade network with producing private label goods. A number of indicators, many of which are also presented as a phase space, can be used to describe this system.

The management of a trade network node impacts the state of the system. Its task is to develop a management decision that ensures the maximum of the relevant criteria for evaluating performance indicators after the planning period, which is divided into n periods. In turn, the initial state is also determined at the level of a trade network node. The difference is that the area of acceptable values for the phase space of the node is determined by the top management of the whole network. Despite the two tasks being structurally similar, the methods for solving them are different.

Choosing the Method for Mathematical Modeling on the Basis of Digital Twin

The models used for solving economic problems, even the simplest approximately reflecting real-life processes, are for the most part non-linear. The problem formulated above belongs to the class of multi-dimensional, non-linear problems. Its solution is based on searching for an optimal set of management decisions. During the research process, the discrete maximum principle known as Pontryagin's principle was used. Its use in the developed model was due to the search for an optimal solution for the whole system of businesses included in the trade network. An addendum was developed based on the resulting solution using another method - the principle of dynamic programming. This algorithm is implemented step by step in the following way (using the introduced symbols).

The model formalized above of a process taking place at the level of a trade network node creates the task of searching for a sequence of management decision vectors $\overline{U_i} = (U_{i1 \ldots\ldots} U_{im})$ for $i = 1 \ldots n$. A visual representation of the process is seen in Figure 2.

$$\overline{U}_i$$

$$S_{i-1} = \sum_{k=1}^{m} Z'_{i-1\,k} \rightarrow \boxed{S_{i-1} \times \overline{U}_i \rightarrow F(\overline{S}_{i-1}, \overline{U}_i) \rightarrow \overline{Z'_i}} \rightarrow S_i$$

FIGURE 2
FLOWCHART OF MANAGEMENT DECISION MAKING

Where the formula $S_i = S_i(S_{i-1}, U_i)$ characterizes the dependences of the transition from state S_{i-1} to S_i; vector elements F reflect the network response $Z'_{ik} = F_k(\overline{Z}_{i-1}, \overline{U}_i)$ when $k = 1 \ldots m$

Current costs are equal $\phi_k(z) \le z$ for each of the $k = 1 \ldots m$ correspondingly equal for each enterprise as a node of the trading network: $F_k = f_k(S_{i-1} \times U_{ik}) + \varphi_k(s_{i-1} \times U_{ik})$, and the corresponding elements U_{ik} for all $k = 1, 2, \ldots, m$ are located on a standard simplex $m-1$.
Application of the Pontryagin maximum principle allows obtaining the necessary set of managerial decisions U^*_i .

Computer programming and the small amount of computing and the simplicity of the calculations using the formulas above are positive qualities of this method.
Now the value of the cumulative criterion W of the given formula (3) needs to be combined with the calculated values for each interim period W_i for $i = 1 \ldots n$: $W = \sum_{i=1}^{n} W_i$. In order to solve this problem, Bellman's principle or dynamic programming is used.

It should be noted that the method for dynamic programming is laborious in its computing process. However, the use of modern processes and specialized packages of applications and add-ons makes it possible to pass the cumbersome calculations onto the computing power of a computer.

Market Uncertainty for the Digital Twin

Real commercial activities always take place in conditions of market uncertainty. This is reflected in the randomness of all the features such as the volume of demand for the positions of the assortment matrix, seasonal irregularity, and adverse effects of the competitive environment. During the management decision making process, managers have only the average indicators. The problems and their solution considered above provide for the deterministic nature of the variables. In reality, each of the nodes of the trade network is made up of a number of divisions, the performance indicators for each of which area stochastic value. However, management decisions are made at the top level of management, and individual indicators are already added up and averaged. In this way, it is necessary to solve the problem of mathematically modeling the level of a trade network node, which is stochastic.

Digital Twin Decision Making Under Uncertainty

With the purpose to create the stochastic problem for managing a trade network node, the formalisms given above are used. Here the commercial network is presented as the system S, subject to stochastic factors. The set of management decisions U applied to the system S is evaluated by the criterion W. Here, its value is $W = \sum_{i=1}^{n} W_i$, where W_i is the evaluation of the criteria in each of the interim periods which the planning horizon is divided into, where $i = 1...n$. However, in a stochastic problem, the value of W is not fully determined by the chosen U and the initial state S_0. The task of management is to determine the set of decisions U_i^* for $i = 1...n$, which brings the system S to the state S_{FIN} over n interim periods and ensures the maximum criterion W (Krasnov, Sergeev, Zotova, *et al.*, 2019). Since the problem is stochastic, it is possible to speak only about its average value $\overline{W}$. Expressing only through the expected value gives $\overline{W} = M\left[W\right]$.

Solving the Stochastic Problem

The formulated problem is solved by the method of dynamic programming. At the same time, the goal of the solution is the sequence of vectors $\overline{U}^*$ for $i = 1...n$, which are a set of management decisions from the management of a trade network. The procedure for searching for $\vec{U}_i$ is determined by the criterion $\overline{W}$, which needs to be maximized.

Generally, the solution takes place for the i-th interim period from the condition of finding the trade network system in the previous state S_{i-1}, which, together with a set of management decisions U_i and stochastic distribution dependent on S_{i-1}, fully determines the following state of the network S_i. In these conditions, the expected value is equal to $\overline{W_i}(S_{i-1}, U_i) = M\left[W_i(S_{i-1}, U_i)\right]$.

This stochastic problem differs from a deterministic one in that the set of optimal management decisions is also of a random nature. It is chosen already from the condition of trade network activities during uncertainty. At that, the calculation is reduced to a set of management decisions, which are optimal in the state of the system after the random result obtained in the previous interim period. This is consistent with the pattern of management decision making using feedback in connection to the current state.

The difficulty of calculating a stochastic problem is higher when compared to a deterministic problem. However, by using software add-ons, the computing load falls on processor power. When using modern computers to create an expert system, the management of a trade network can use scientifically based solutions in their activities in real time.

Solving Example

To demonstrate the above technique, we will give an example of calculation. For clarity, consider two enterprises - nodes included in the trade network. Let us denote the trade network node as TNN1 and TNN2. Let us compare the use case of the proposed algorithm based on the dynamic programming method with traditional planning based on a moving average. The initial data are the amount of funds distributed between the nodes, the return on assets of each of them and the current costs. The planning horizon is 12 months. Trade enterprises with various indicators of trade activity, which are determined by the socio-demographic landscape of their location, are considered. Figure 3 shows the results of calculations according to the given algorithm on the phase plane.

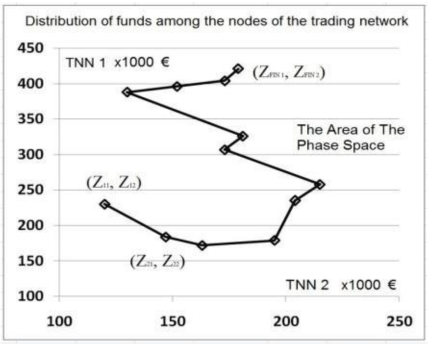

FIGURE 3

THE PROCESS OF CALCULATING THE PLANNING ORDER USING DYNAMIC PROGRAMMING

In order to be able to assess the effect of the dynamic programming method, Fig. 4 shows a comparison of the final economic efficiency in comparison with the moving average method. From a comparison of the two diagrams, you can see that the computer chooses already at the first stage to redistribute funds between TNN1 and TNN2.

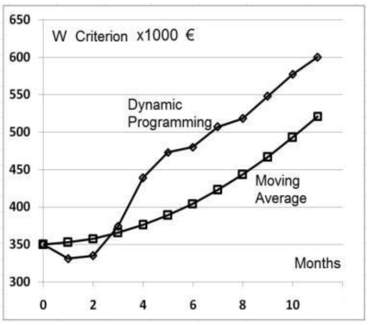

FIGURE 4

CALCULATION OF THE DEPENDENCE OF MANAGEMENT QUALITY CRITERION W

With further steps, the calculated values are constantly adjusted, which makes it possible to increase the total indicator of the economic efficiency of joint planning.
The calculation was carried out for a short horizon and at a small number of nodes for clarity. For real practical calculations on a computer, the dimension of the vectors, the determined number of nodes that make up the trading network and the duration of the planning period do not matter.

DISCUSSION

Points for scientific discussion could be considered as follows. Virtually, real-time information processing technology allows real-time monitoring of commodity design, raw material purchasing, production, transportation, storage, distribution and sale of semi-products and products, returns' processing and after-sales service. We agree with the fundamental approach (Tekinerdogan & Verdouw, 2020) assuming that developing digital twin-based systems requires a multidisciplinary approach.

Issues for the Scientific Discussion Regarding the Optimization

The methodology offered by the authors for algorithmizing a digital twin of a trade network is based on a step-by-step solution of the optimization problem. First, the optimization problem is solved at the level of a trade network business based on Pontryagin's principle, and then based on the method of Bellman's dynamic optimization. This paper shows that mathematical formalisms for the level of a trade network node are similar to those common for the entire system examined, covering the whole trade network. Each of the businesses also has an internal structure. The management of the trade network node influences the state of the system. The area of acceptable values for the phase space of the node is determined by the top management of the whole network.

Taking into account the proposed definition of the digital twin as a software implementation of the algorithm for processing a set of data arriving online, the future scientific research can be in the development of scientific discussion on questions of optimization methods for the development of a trade network.

In addition, the special features of digital twins from the economical point of view could be discussed regarding the economic nature of the digital twin ranging from the aging of a digital object, the structure of costs for production of digital twin in the form of a flow of goods and dividing them into constants and variables, and, finally, the possibility of leasing as well as the open innovation concept.

The Features of the Digital Twin Being Discussed from the Open Innovation Concept Point of View

Some methodological and philosophical issues could arise regarding the nature of the digital twin from the open innovation theory point of view. Christensen et al. (Christensen et al., 2005) address how the open innovation concept can be analyzed from an industrial dynamics perspective, considering the specific measures that different companies take to manage open innovation from the standpoint of their differential position within the innovation system in question, the nature and stage of maturity of the technological regime, and the particular value proposition pursued by companies focusing closely on the complex interplay between technology entrepreneurs and incumbents.

A concept model of open innovation built up in (Yun et al., 2020) is intended for exploring the existing open innovation channels, which can be useful to motivate engineering research increasing the development of open innovation and new open business models. In his article Chesbrough notes that the key trends in the development of open innovation are related to the field of digital transformation (Bogers et al., 2019). Open innovation can create an ecosystem in which people and organizations interact. It involves business models—the logic of creating and capturing value—that dynamically transcend organizational boundaries within that innovation ecosystem (Bogers et al., 2018). The development of digital technologies allowed small organizations to rapidly increase their influence in the market. Such innovations were defined by Clayton M. Christensen as disruptive. With his term-disruptive innovation, Clayton M. Christensen reflected the complexity and inconsistency of innovation in General (Christensen et al., 2018). The digital twin models are parcel parts of digital trading ecosystems involving physical distribution channels.

Discussing the dualism between the real and virtual world in the concept of a digital twin has led to some interesting issues to be considered be the authors in general.

The Theoretical Features of the Digital Twin Being Discussed from the Economical Point of View

From the point of view of accounting for a digital object, we can consider the aging of a digital object. If you decompose the processes that occur with a digital object in time, we can apply the same approaches to it as to any physical object: Aging and degradation.

Degradation in this case refers to the gradual loss of relevance of a digital object associated with changes in the physical object (aging and degradation of its elements, intentional and unintentional changes in the physical object), which are not recorded or not fully recorded in the digital object. The degradation can be compensated by well-established facility monitoring system (sensors, IoT), debugging, information flow "physical world "digital world" coming from the sensors and manually making changes in a physical object (e.g. repair, maintenance, replacement of components, changing the destination premises, etc.). We can say that the quality of installed sensors and IoT elements, along with the quality and “reasonableness” of sensor installation, while increasing the level of adequacy of the mathematical model of objects and processes in the digital world, reduces the degree of degradation to which the central information system is subject.

By aging, we mean the aging of digital technologies used in a digital object. As a rule, improving the quality of products on the market leads to an increase in the minimum requirements for it on the part of consumers and legal regulation, which implies the "aging" of the product. Similarly, digital technologies that are the market leader today may be "old" tomorrow, in which case the market can improve the quality of data collection about a physical object, the speed and frequency of data transmission, the accuracy and detail of data about objects and processes in the physical world. In contrast to degradation, the aging of a digital object can only be compensated by post-sale maintenance of the central office.

The economic approach to developing a digital twin involves studying the structure of costs and dividing them into constants and variables.

The cost structure for the production and improvement of the digital twin includes the cost of paying for specialized software, the salary of developers, the purchase and installation of sensors and elements of the Internet of Things. Improving and maintaining the operation of the digital twin also requires the cost of specialists ' salaries, maintenance, purchasing materials for sensors, and maintaining the information model.

The division of costs into constants and variables when working with a digital twin is carried out from the time position. Costs can be divided into fixed-the primary development of a digital object and variable-maintaining the relevance and performance of a digital object. In addition, as part of the initial development of a digital object, variable time costs are also laid down, depending on the duration of work on the information model. By volume, we can also divide the costs into fixed costs-platform development and variables, as a derivative of the total cost, depending on the volume and complexity of the physical object, the number of elements included in it, the level of detail of the digital object, the number of sensors and IoT (Internet of Things) elements installed on the physical object, the volume of data collected and transmitted. Production of digital twin in the form of a flow of goods is a very interesting issue for scientific research including both theoretical and practical aspects of studying the economical features of such a new element of digital economy.

Planning the streaming production of digital twins requires the initial development of a platform for creating digital twins. In the presence of such a platform, the flow of digital objects will objectively correspond to the flow of physical objects undergoing digitalization.

The authors find it interesting to show the absence of the possibility of occurrence by analogy with ordinary goods in their physical execution. We can have «surpluses of unrealized products» and «stocks of finished products», since each digital twin corresponds to a certain physical object. The exception is the development of digital twins for typical construction or production, in which case the stocks of finished products, as well as the surplus of unrealized products will be embodied in the form of created digital models, for which a physical prototype has not yet been created, but since we put the dualism between the real and virtual world in the concept of a digital twin, such surpluses could not strictly be called digital twins. In addition, in the form of surplus finished products or stocks of unrealized products, a part of the digital twin can be implemented in the digital field, but not related to its physical prototype (including in the absence of the corresponding part of the physical object). Such surpluses and reserves can be considered realized only after the installation of bidirectional information flows between the physical and material components, which results in the ability of the digital model to bring material benefits to the owner of the physical object.

Leasing is also possible and involves temporary partial or full transfer of rights to use, view and edit digital object (in this case, it is necessary to divide the components of the digital object on the main, unavailable to the tenant to edit, and time available to the lessee for editing).

CONCLUSION

The development of the physical distribution channels within the trading network, the digitalization of the delivery of goods and services, the transfer of trade and payment transactions to the digital space provide both means and tools for creating scientifically proved digital trading ecosystems.

In this article the suggested meaning of the physical distribution digital twin model within the trade network could be as follows: it is a software implementation of the algorithm for processing a set of physical distribution data arriving online and at each moment of time reflecting the activity of the considering trade network.

The scientific contribution of this work lies in the use of the mathematical apparatus for finding optimal solutions for the digital twin of the trading network. Until now, there was no technical base and massively available technologies that would allow obtaining real-time data on the current state of such a complex economic object mentioned above. The proposed approach allows using the input data flow as arguments of the functions of the mathematical model and calculating the optimal strategy for the management of a trading network.

Online information on all stages of transactions and interaction with expert systems based on proven algorithms set the foundation for making management decisions as a result of the program processing of data obtained from the digital seamless M2M interaction. At the same time, the target indicators of trade activity are checked against optimality criteria for conducting business. The possibility of conducting stress scenarios, innovative IT solutions, sound business diversification, and adaptation to the variable components of the digital ecosystem landscape, reduces risks when conducting business.

Developing scientifically based algorithms of a digital ecosystem is relevant since the process of growing commercial networks in most developed countries occurs for regional and transnational expansion. Due to the fact that the processes examined take place in the context of the negative background of the economic situation, optimization achieved through the method of mathematical modeling is becoming a driver in the competition. Currently, there is a global trend of business consolidation into trade structures (networks). This trend of forming trade networks concerns not only the retail sector but also the service sector. Its appearance has been especially clear over recent years influenced by the digital transformation. The line between digital sale of goods and service implementation is erased since global shifts of the whole trade landscape have taken place. The connection between a physical object and its virtual model is at the base of the concept of creating a digital twin, which includes optimization models whose data is the function arguments for a mathematical model of a digital twin.

CONFLICT OF INTEREST

The authors confirm that there is no conflict of interests to declare for this publication.

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