MODEL FOR FORMING OF OPTIMAL CREDIT PORTFOLIO OF COMMERCIAL BANK

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ABSTRACT

A methodological approach to the optimization of a commercial bank's credit portfolio is proposed based on Markowitz economic and mathematical optimization model. The economic and mathematical model has been formed, relating to optimization of the bank's credit portfolio based on balancing the factors of maximum profitability and given level of credit risk. A methodical approach was formed to form a credit portfolio by types of credit services and the level of profitability of each of them in the total amount of credits granted.

Keywords: Markowitz model, Credit portfolio, Commercial bank, Covariation

INTRODUCTION

Each commercial bank has as its main goal an increase in profit margins while simultaneously fixing or reducing the cost of raising funds. This is possible by reducing the volume of passive operations, or by rational allocation of borrowed funds in active operations of the bank. Therefore, the main task should be the effective allocation of resources precisely in credit operations. For this, it is necessary not only to qualitatively analyze the credit portfolio of the bank, but also to form its structure in such a way as to maximize profitability for each type of credit operations (products), including the risk of non-repayment of credit by borrowers.

LITERATURE REVIEW

To analyze the credit portfolio, the following components are used: assessment of the quality of credits constituting the portfolio; determining the structure of the portfolio as a function of credit quality and determining the structure as a function of its dynamics (Casu et al., 2012; Drobyazko S. et al., 2019a, 2019b). As Mishkin and Eakins (2006) note, a bank typically uses several types of credit products: overdrafts, credit lines, term credits, aval credits, interbank credits and the like. The main factor is the value of the portfolio, as the total value of all components of the credits actually granted for a certain time (Rose, 2002).

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METHODOLOGY

Let x_i be the share of capital spent to provide the i^th loan. All allocated capital is taken as one. Let the d_i profitability in percentage of annual loans of the i^{th} type per one monetary unit. Find the yield of the entire portfolio d_p. On the one hand, in a year the portfolio capital will be equal to $1 + d_p$, on the other hand, the value of loans of the i^{th} type will increase from x to $x_i + d_i x_i$, so that the total value of the portfolio will be $\sum x_i + \sum x_i d_i = 1 + \sum_i x_i d_i$. Comparing two bonds for the portfolio value, we get:

$$1 + d_p = 1 + \sum_i x_i d_i ; d_p = \sum_i x_i d_i \quad (1)$$

Therefore, the task of raising portfolio capital is equivalent to the similar problem of portfolio profitability, expressed in terms of loan yields and their share of the formula (1). Let m_i, σ_i be the average expected return and the mean square deviation of this random return, that is, $Mm_i = M[d_i]$ is mathematical expectation of profitability and $r_i = \sqrt{v_{ii}}$, wherein v_{ii} is covariance of the i^{th} profitability. The mathematical expectation of a discrete random variable is called the sum of the product of all its possible values on their probabilities (Fama & French, 2004). Covariance is the degree of association between two data ranges $ov(X, Y) = \frac{1}{n} \times (x_i - u_X) \times (y_i - u_y)$.

We will call m_i, r_i, respectively the efficiency and risk of the i^{th} loan. Denote by v_{ij} the covariance of the yields of loans of the i^{th} and j^{th} types (or the correlation moment K_{ij}). Because the yield of the component portfolios of securities is accidental, the yield of the portfolio is also a random variable. There is a mathematical expectation of a portfolio's profitability $M\left[d_p\right] = x_1 M[d_1] + \cdots x_m M[d_m] = \sum_i x_i m_i$, denote it by m_p. There is a variance in portfolio returns $\left[d_p\right] = \sum_i \sum_j x_i x_j v_{ij}$. Just as for loans we call m_p- portfolio efficiency, and the value $\delta_p \sqrt{D[d_p]}$ is the risk of portfolio r_p. Typically, the variance of portfolio returns is denoted by V_p. Variance of Discrete Random Value- Mathematical Expectation of a Square Deviation of a Random Value from its Mathematical Expectation: $D(X) = M[X - M(X)]^2$.

Each bank is faced with a dilemma: one wants to have the effectiveness of the credit portfolio more and the risk less. The model of the optimal Markowitz portfolio, which ensures minimal risk and a given profitability, has the form:

$$\begin{cases} v = \sum_i \sum_j x_i x_j v_{ij} \to min \quad \text{m and n} \\ \sum_i x_i d_i = m_p \\ \sum_i x_i = 1 \end{cases} \quad (2)$$

Need to determine: $x_1, x_2 \ldots\ldots x_n$. An optimal portfolio of Markowitz (1952) maximum profitability and a given (acceptable) risk r_p can be represented as:

$$\begin{cases} m_p = \sum_i x_i d_i \to max \\ \sum_i \sum_j x_i x_j v_{ij} = r_p \\ \sum x_i = 1 \end{cases} \quad (3)$$

The arithmetic average yield of the i^{th} loan is calculated by the formula:

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$$d_i \approx \frac{1}{N}\sum_{k=1}^{N} d_{ik} \quad (4)$$

Covariance or correlation of yield of credit products:

$$V_{ij} = M\left\{\overbrace{(d_{ik}-d_i)}^{\Delta_{ik}}\cdot\overbrace{(d_{jk}-d_j)}^{\Delta_{jk}}\right\} \approx \frac{1}{N}\sum_{i=1}^{N} \Delta_{ik} \times \Delta_{jk} \quad (5)$$

Wherein, Δ_{ik} Δ_{jk} are deviation of the Ith and j^{th} yields from the arithmetic average yield.

RESULTS AND DISCUSSION

For comparison, consider the model of optimal formation of Tobin investment portfolio. The Tobin model, like the Markowitz model, aims at determining the optimal portfolio and risk for them (Gomes & Khan, 2011). Tobin portfolio of minimal risk is expressed as follows:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j v_{ij} \rightarrow min \\ x_0 m_0 + \sum_{i=1}^{n} x_i m_i = m_p \\ x_0 + \sum_{i=1}^{n} x_i = 1 \end{cases} \quad (6)$$

Where:

m_0 – effectiveness of risk-free investment;
x_0 – share of capital invested in risk-free investment;
x_i, x_j – the share of capital invested in investments of i-th and j-th types;
m_i – mathematical expectation (arithmetic average) of the i-th investment return;
v_{ij} – correlation point between the investment performance of i-th and j-th types;

Tobin portfolio of maximum efficiency is expressed as follows:

$$\begin{cases} x_0 m_0 + \sum_{i=1}^{n} x_i m_i \rightarrow max \\ \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j v_{ij} = r_p \\ x_0 + \sum_{i=1}^{n} x_i = 1 \end{cases} \quad (7)$$

Where, r_p – portfolio risk.

When choosing a model to calculate the optimal credit portfolio, we use the Markowitz optimization model.

Considering that today commercial banks use rather rigid assessment of credit risks and financial position of borrowers, we can say that the weighted average risk on credit operations is relatively constant. Therefore, the value of credit risk in calculating the Markowitz model is assumed to be equal to a constant value of approximately 5-6% (Dobler, 2005). Let`s consider forming a credit portfolio for a time lag of 27 periods (Table 1).

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TABLE 1
THE YIELD OF THE CREDIT PORTFOLIO BY TYPES OF CREDIT OPERATIONS

Time lag	Overdrafts	Credit lines	Term credits	Aval credits	Consumer credits
1	0.00518	0.00595	0.00139	0.00061	0.00293
2	0.00498	0.00623	0.00143	0.00081	0.00269
3	0.00550	0.00648	0.00170	0.00071	0.00316
4	0.00580	0.00698	0.00162	0.00091	0.00334
5	0.00616	0.00704	0.00175	0.00110	0.00342
6	0.00595	0.00759	0.00180	0.00134	0.00318
7	0.00584	0.00836	0.00191	0.00130	0.00363
8	0.00569	0.00811	0.00177	0.00124	0.00329
9	0.00692	0.00759	0.00171	0.00113	0.00374
10	0.00537	0.00800	0.00193	0.00105	0.00312
11	0.00524	0.00723	0.00178	0.00100	0.00284
12	0.00541	0.00694	0.00166	0.00069	0.00262
13	0.00555	0.00664	0.00141	0.00072	0.00269
14	0.00560	0.00689	0.00150	0.00072	0.00281
15	0.00584	0.00716	0.00151	0.00069	0.00298
16	0.00595	0.00737	0.00157	0.00081	0.00318
17	0.00687	0.00735	0.00163	0.00088	0.00326
18	0.00746	0.00757	0.00172	0.00084	0.00345
19	0.00735	0.00792	0.00178	0.00126	0.00336
20	0.00655	0.00748	0.00208	0.00139	0.00329
21	0.00555	0.00775	0.00228	0.00138	0.00311
22	0.00594	0.00722	0.00222	0.00130	0.00301
23	0.00652	0.00703	0.00200	0.00110	0.00275
24	0.00744	0.00672	0.00187	0.00096	0.00282
25	0.00791	0.00691	0.00163	0.00079	0.00321
26	0.00790	0.00735	0.00177	0.00086	0.00319
27	0.01012	0.00768	0.00187	0.00100	0.00321

To determine the average yield of each type of credit, we use formula (4). The average yield on overdrafts is 0.00632, on credit lines- 0.00724, term credits- 0.00175, aval credits- 0.00098, consumer credits- 0.00312. To calculate the covariation, we apply formula (5). The calculation of the covariation of the general sample of the yield of credit transactions is shown in Table 2.

TABLE 2
COVARIATION VALUES OF THE GENERAL YIELD SAMPLE, V_{ij}

Bank products	Overdrafts	Credit lines	Term credits	Aval credits	Consumer credits
Overdrafts	0.0000012998	–	–	–	–
Credit lines	0.0000001676	0.0000003200	–	–	–
Term credits	0.0000000465	0.0000000706	0.0000000513	–	–
Aval credits	0.0000000248	0.0000000952	0.0000000443	0.0000000584	–
Consumer credits	0.0000001132	0.0000001005	0.0000000146	0.0000000314	0.0000000830

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Having received the necessary data V_{ij} – covariation, d_i – average yield for each type of credit (Dell'Ariccia, 2001), it is possible to form a target function for calculating the Markowitz optimization model:

$$M_p = \sum xj \times d_{ia} \rightarrow \infty \qquad (8)$$

Where,
X_1 = share of overdrafts in the structure of the credit portfolio;
X_2 = share of credit lines in the structure of the credit portfolio;
X_3 = share of term credits in the structure of the credit portfolio;
X_4 = share of aval loans in the structure of the credit portfolio;
X_5 = share of consumer credits in the structure of the credit portfolio.

We construct restrictions on the optimization model.

Restriction 1: $\sum \sum x_i \times x_j \times v_{ij} = r_p$,

Where, r_p – permissible permanent risk equal to ≈ 5%

Restriction 2: $\sum x_{ij} = 1$from the system of portfolio optimization (3).

To solve the Markowitz optimization model, we substitute the value in the cell "target function" M_p**,** determine the search values: X_1, X_2, X_3, X_4, X_5 and introduce the limitations above. After entering all the necessary data, we calculate the objective function and X_i. The results obtained are shown in Table 3.

TABLE 3
RESULTS OF CALCULATIONS OF THE TARGET FUNCTION AND XI ACCORDING TO MARKOWITZ MODEL

X_n	Markowitz model	Bank credit portfolio structure	Deviation
X_1	73%	55.3%	17.7%
X_2	20%	35%	15%
X_3	3%	5.1%	2.1%
X_4	2%	3.5%	1.5%
X_5	2%	1.1%	0.9%

Thus, the structure of the bank's credit portfolio will be optimal if the distribution of the credit portfolio is in the ratio: 73% should be diverted to overdraft; 20% - in credit lines; 3% in term credits; 2% - in aval credits; 2% - in consumer credits.

CONCLUSIONS

An economic and mathematical model of credit risk decrease is proposed using Markowitz calculation model. The limits set by the bank's policy on credit portions in the credit portfolio are minimal in the model. That is, when cancelling them, the advantage will be completely on the side of overdraft credits. In order to increase the income from credit operations, the bank now needs to divert credit to overdraft credits because their average yield is higher than all other credit operations. The calculation of the credit portfolio

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optimization model revealed that the amount of credit risk is too small from the standard rate and is 0.000031%. With this amount of credit risk, the bank is able to credit the most borrowers.

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