TEACHING SYNERGISTIC INTEGRATION OF ECONOMICS AND MATHEMATICS

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ABSTRACT

The purpose of this analysis is to present a Synergistic Integration Method (SIM) of enhancing higher level thinking and obtaining robust applications of numerous mathematical applications through problem design and presentation. This technique involves constructing problems that embed mathematical constants that are universally encountered across the arts and sciences. Although this method has been developed for use in economics, it can be applied in any mathematically-based discipline. Once the concepts are manifested throughout the problem specification and solution, the results are used to present applications from a variety of disciplines that extend well beyond economics. This methodology has a big payoff for instructors and for students by sharpening analytical skills, by increasing retention of material from broad curricula, and by enhancing creativity and awareness.

INTRODUCTION

A job worth doing is a job worth doing well. Most concepts in economics are primarily taught by presenting and solving a problem that is relevant to the issue of concern. When equations, numbers, and graphs are used as examples, they simply consist of arbitrary numbers and curves that are chosen with no plan or design other than to contain a solution that falls in some desired range. Once the example achieves the minimum objective, students move on to the next topic and the task is completed. The job is considered done, but it is not done nearly as well as it could be.

This paper presents a Synergistic Integration Method (SIM) that views every example not only as a utilitarian exercise, but also as a potential work of art. This method extends the teaching of any concept well beyond a given problem's initial seemingly narrow focus, and moves toward finding an additional inherent beauty through an internally forged esoteric design that bursts forth a wealth of unexpected applications that cross over all disciplines. Students' learning truly becomes an adventure. The diversity of this method ensures that the instructor can draw from areas targeted such that each student finds the study more meaningful, relevant, and applicable to his or her own interests.

METHOD DESCRIPTION

The SIM begins by embedding fundamental mathematical concepts within each example. This creates a layered esoteric, or hidden, design. It creates a story within a story. Only the students who recognize the mathematical constants will be able to understand both stories. Using this procedure, the problem may contain multiple layers whereby only the most knowledgeable students will fully perceive all that is hidden within the problem. As the problem is presented and solved, the instructor can share the dual ideas within the problem, and perhaps leave some of the other concepts as a puzzle or exercise for the student to discover. As will be shown below, students may have a partial perception of the underlying concepts without fully understanding why.

Finally, the instructor can present various applications of the mathematical concept in the non-economic areas where they used. Students learn not only economics but also a range of other interesting integrated topics as well. This binds the students' entire curricula together, and creates a synergy that arises due to the insights gained through the exploration of the common strands of various applications that share an underlying kernel. Without this nexus, these additional insights would otherwise be unobtainable.

The author has applied this method in a variety of economics and mathematics courses at the high school, undergraduate, master's, and doctoral level for over 20 years, and has found it adds greatly to the educational experience. It is based on an ancient technique of educating initiates that was used by the ancient Egyptians, Pythagoreans, and many others from unknown remote antiquity up through the present.

EXAMPLES AND ANALYSIS

This section demonstrates the method through the use of four examples. For the first example, consider a microeconomic problem of determining the profit maximizing quantity of production for a monopolist.

Example 1: Consider a monopolistic producer that faces a cost function and an inverse demand function given respectively by C = 480,000 + 1947.1Q and P = 5159.7324 - .712463335 Q. Find the profit-maximizing quantity, price, and profit.

Figure 1

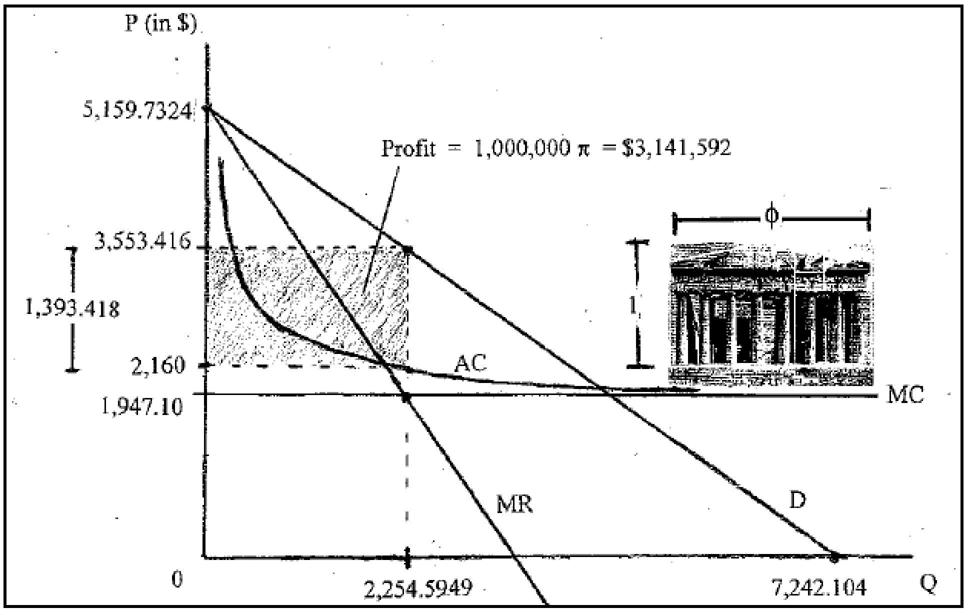


Figure 1 shows the (inverse) demand (D), marginal revenue (MR), marginal cost (MC), and average cost (AC) functions for this situation. The profit-maximizing quantity of production is 2,254.5959 units. Although firms generally must produce whole numbers of units, the numbers should be left as precise decimals or fractions in order to catch the full significance of the underlying values. The profit maximizing price is $3,553.416. Although prices are usually stated to the nearest cent, it is often insightful to round the price to the third decimal place, as is routinely done with gasoline prices, in order to achieve more precision in the results.

At the profit-maximizing price and quantity, the average cost (AC) is $2,160. The moon is exactly 2,160 miles in diameter! A figure of 2,160 years also commonly used as the ideal number of years for the earth to move through one astronomical age in the cycle of precession of the equinoxes. The average profit per unit sold is $3,553.416 - $2,160 = $1,393.418, and the total profit to the nearest dollar is given by ($1,393.418)(2,254.5459) = $3,141,592. This number is 1,000,000 p! This is the area of the shaded rectangle in figure 1.

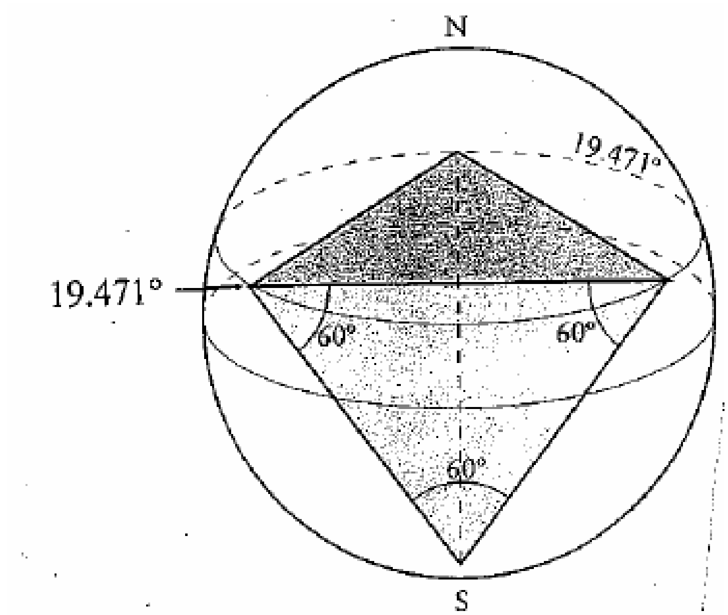
This example thus provides the decimal harmonics of two widely occurring mathematical constants, 216 and π. The frequency 216 Hertz is an octave below a tuning standard of A = 432 Hertz, which is the tuning of the Stradivarius violin, and the tuning standards that many musicians advocate. The Bose Corporation sets the clock time to display 4:32 on the promotional materials for its wave radio, reflecting this tuning standard.

There are numerous other applications that could be pointed out. For example, students can be asked to find the circumference of the moon using the formula circumference equals π times diameter, so that C = (π)(2,160) = 6,786 miles. This links the two constants together. Students thus get a lesson in economics, astronomy, music theory, and geometry. But that isn't all, because there are two other gems hidden in the problem.

Examine the shape of the rectangular profit region. The ratio of the base to the height of the rectangle is 2,254.5949 / 1,393.418 = 1.61803. Voila—this is the golden number ϕ to 5 decimals places! The number f is precisely equal to (1 + Ö5)/2, and it is the limit of the ratio of the numbers in the Fibonacci sequence, where each successive adjacent numbers are added together to give 1, 1, 2, 3, 5, 8, 13, … This also means that the profit region forms what is referred to as a golden rectangle. This is considered to be the most ascetically pleasing rectangular shape for humans, and it has been designed into the great architectural structures of the ancient Egyptians, especially including the Great Pyramid at Giza, the Osirion at Abydos, and the temple at Luxor (Hudgins, 2003, Lawlor, 1998). It was commonly used by ancient Greeks, such as in the façade of the Parthenon, and has been employed by many countless others across the globe from ancient times throughout the present. Students get a lesson in art and architecture by examining this aspect of the problem. But that is not the end.

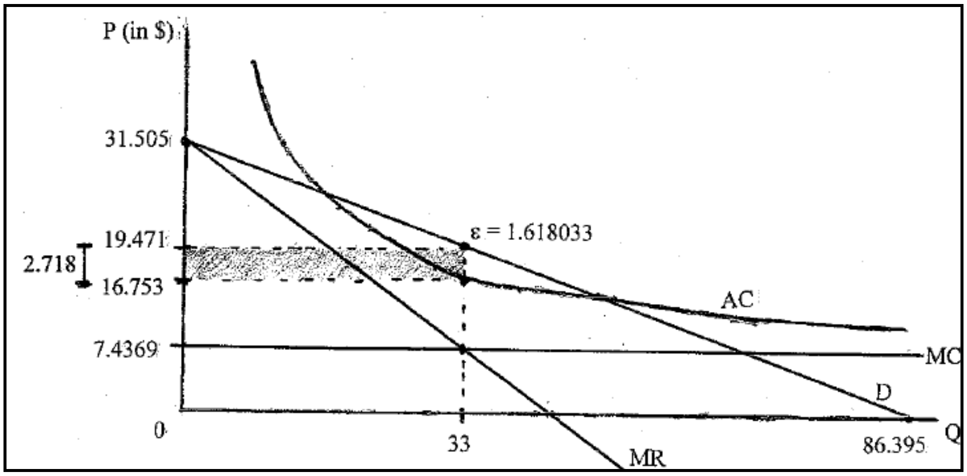
The marginal cost (MC) is $1,947.10. Figure 2 shows a regular tetrahedron, which is the simplest of the five Platonic solids, inscribed within a sphere. When the apex of the tetrahedron is at the North Pole, the three base corners will be tangent to the sphere at 19.471 degrees south latitude. Similarly, if the apex is placed at the South Pole, then the three base corners will be tangent at 19.471 degrees north latitude. Thus, the marginal cost is equal to the decimal harmonic 100 t, where t is the circumscribed tetrahedral constant 19.471.

Figure 2



The largest volcanoes on earth occur in Hawaii, whose Big Island contains latitude 19.471. Olympus Mons, on the planet Mars, which is the largest volcano in the solar system, also resides at between 19 and 20 degrees, as does Jupiter's big red spot. Mexico City lies atop the old Aztec capital of Tenochtitlan at latitude 19.47 degrees. The state of Israel, which has a 2-dimensional version of this interlocking double tetrahedron on its flag, was enacted and placed in existence in 1947 and 1948. India's independence occurred in 1947, and the U.S. Air Force became an independent service in that year. Much of the post-war economic data begins in 1947. Thus, students can use these and other applications to learn lessons in geometry, geography, astronomy, and history.

Figure 3



Example 2: Consider a monopolistic producer that faces a cost function and an inverse demand function given respectively by C = 307.44 + 7.4369Q and P = 31.505 - .36464Q. Find the profit-maximizing quantity, price, and profit.

Figure 3 shows the graph for this situation. The profit-maximizing firm will produce a quantity of 33 units and sell them at a market price of $19.471. Here is the circumscribed tetrahedral constant again, but now it is augmented with a new fact. The constant t actually derives geometrically from the fact that $sin^{-1}(1/3)$ = 19.471 degrees. Since 1/3 = .333…, the constants 33 and 19.471 are linked. There are many applications of this, but students get a bonus whenever the problem contains both figures.

In this problem, the students get other bonuses, too. The price elasticity of demand is ε = (1/.36464)(19.471)/(33) = 1.618. Voila—there again is the golden section! And the students are sure to notice these numbers and their applications the second time around. Also, the average profit is equal to price (P) minus average cost (AC), so that $19.471 – $16.753 = $2.718. This is the widely used exponential constant *e* = 2.718, given to 3 decimal places. This constant appears throughout every field of mathematical application, especially in problems of growth and decay.

Example 3: A family has spent \$19,471 to purchase a mutual fund to save for a daughter's college education. She plans to attend college in Hawaii. They expect the fund's annual rate of return to be steady at about 5.95%. If this holds true, how many years will it take for the value of this investment grow to \$60,000?

This example comes from statistics, algebra, and financial mathematics. The solution of this problem is 19.471 years, and it is found by solving the following equation for n: $60{,}000 = 19{,}471\ (1 + .0595)^n$. The solution to this problem is the circumscribed tetrahedral constant t, and the principle of the investment in this example is 1,000 t. The students can make the connections that Hawaii contains the latitude 19.471 degrees north. The Hawaiian Islands also contain latitude 21.6, which runs by Honolulu, the birth place of U.S. president Barack Obama. This number is 216/10, where the number 216 and 2,160 were previously presented. The location of the college and monetary amounts can easily be altered, or used as they are in order to express other geographical and political lessons. For example, the president lives in Washington, D.C., which lies at a latitude that is exactly $2t$, or (2)(19.471), which is between latitudes 38 and 39 degrees north.

The solution setup also contains the figure (1 + .0595) = 1.0595. This is the four decimal approximation of $2^{(1/12)}$, which is the incremental ratio between the frequencies of adjacent notes in an equal temperament tuning scale, which is used on most modern musical instruments. The instructor can explain equal temperament within the lecture, or outside of class to just those students who want to know more detail. This example thus allows for a lesson in geometry, geography, astronomy, history, politics, and music theory.

Example 4: A country produces cheese and wine, and it possesses 660.542 hours of labor where its Production Possibilities Frontier (PPF) is given by $256\ Q_C + 243\ Q_W \leq L$, where represents labor measured in hours. Find the slope-intercept form where Q_W is the dependent variable, and state the opportunity cost of producing cheese.

This example appears in microeconomics and international trade. The slope-intercept form of the equation found by calculating the following:

$$Q_W \leq (660.542 / 243) - (256 / 243)\, Q_C,$$

which has the solution

$$Q_W \leq 2.71828 - 1.0535\, Q_C.$$

The intercept of this equation is the exponential constant $e = 2.71828$, given to 5 decimal places. The slope of this equation is 256 / 243 = 1.0535, which is also the opportunity cost of producing cheese. In order to produce one additional unit of cheese, the firm must give up producing 1.0535 gallons of wine. But this number is also the limma, or leimma, (also called a minor semitone) from the Pythagorean scale in music theory, which expresses the difference between the minor third and the tone. This number appears as a frequency in vibratory applications throughout physics (Hudgins, 2003).

CONCLUSION

The Synergistic Integration Method (SIM) presented in this paper integrates different concepts within a carefully constructed framework that consists of a plethora of densely packed examples. Each of these examples tells a story within a story as the students peels back layer after layer of the innards. Students discover new paths for enhancing higher level thinking as they explore the robust range of applications of numerous applied mathematical constants. The subject becomes more of an art—more of a high-stakes game where the reward for spending time with a problem is the sweet fruit of understanding. Synergy results because the combined effect of the SIM exploration results in a journey that produces a much bigger experience and much deeper insight than that which could be obtained by trying to analyze each of the pieces alone in isolation.

The examples above are only a small set of the vast array of potential that this approach offers. Besides the numbers in the examples above, the author has commonly employed either the number or its decimal harmonic of the following: the comma of Pythagoras ($CP = 3^{12}/2^{19} = 1.0136432$); the fine structure constant ($\alpha = 1 / 137.036$); musical pi ($MP = 360/84 = 4.2857$); the radian ($180/\pi = 57.29578$); the square roots of 2, 3, and 5; the sizes, orbital periods, and distances of planets;

geographical coordinates of ancient monuments and modern cities; and many other interesting topics. For example, when working an example regarding current issue facing a country in the field of economic development, the country's geographic coordinates and its landmark dates can be embedded within the problem so that the class can get a fuller grasp on the country in its entirety.

But this approach does not end there. It strives for full integration. Thus, when setting the course schedule calendar for certain lecture topics or term paper due dates, the author has set them to be turned in online by mathematical constants in time. For example, some of these times included: 3:14 a.m. or p.m. (π); 3:33 a.m. or p.m. (1/3); 6:18 a.m. or p.m.; (10/ϕ); 4:18 p.m., which 16:18 hours military time (10ϕ); 7:47 p.m., which 19:47 hours military time (*t*); and so on. These lecture dates or due date times can also be chosen on specific calendar dates, such as the equinoxes and solstices, or dates such as March 14 (3/14 = π).

There also two further aspects that can be integrated: The labeling of the problems and the locations of the problems on the paper, including the margins and the size of the paper. For instance, example 1 above might be placed within a problem set where it is labeled as problem 16, since it contains the constant ϕ = 1.618 (and 16 = 10 ϕ). Example 2 might be numbered as problem 3 or 33, since it contains the number 33 and its inverse *sin* function. The Rhind mathematical papyrus from ancient Egypt places some problems on the page where the dimensions of the worked out problem follow the golden ratio ϕ. For example, in that papyrus, problem 35 and its solution are divided horizontally so that ratio of the dimensions is 1 to ϕ (Schwaller de Lubicz, 2001, pp. 152-3). Why has it been considered acceptable to work out problems and then present them in any manner as long a solution procedure is shown and the answer is correct? This lack of awareness removes the requirement of students to find beauty, art, and pride in all that they do. It loses a sense of purpose.

The economics problems can be typed and placed on the page in the dimensions of the constants that are embedded, such as having a base to height ratio of 1: ϕ (1:1.618), for example. Even the borders on the word processed handout or problem set can be set so that the typing on the page takes up this golden section ratio. Just as with fine Japanese and other cultural food cuisines, the food presentation and atmosphere are considered to be just as important as the food when achieving a fully integrated dining experience.

The more effort that the instructor puts into creatively embedding all of the concepts that are relevant to the students' interests, needs, and learning objectives, the greater is the payoff. It is certain that every problem utilized in teaching cannot be a multi-layered esoteric learning exercise, but the more that this method is utilized, the more aware the students will become as they gain a sense of synchronicity in their experiences. Architecture, art, astronomy, biology, economics, engineering, geography, history, mathematics, music, philosophy, political science, and more are all fused into one magnificent whole. Our objective as teachers is to continually raise our own existence to a higher plane, and to encourage our students to do the same. This is one technique toward following that path.

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